Tie-Breaking with the Single Transferable Vote

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1   Introduction

In tallying the single-transferable vote (STV), ties can occur for several different reasons. With the ERS97 rules [1] for implementing STV, ties can occur when choosing a surplus to transfer (5.2.3), when choosing a candidate to eliminate (5.2.5), and when choosing winners (5.6.2). To illustrate, Table 6.1 shows an example tally with the ERS97 rules. At stage 4, we need to eliminate the candidate with the fewest number of votes, but both C and D are tied for last place.

When ties occur, they need to be broken. One could simply break the tie by lot. However, since there is other information available in an STV count, one can use this information to break the tie. The following are four possible tie-breaking rules.

1. Forwards Tie-Breaking: Choose the candidate who has the most [least] votes at the first stage or at the earliest point in the count where they had unequal votes.

2. Backwards Tie-Breaking: Choose the candidate who has the most [least] votes at the previous stage or at the latest point in the count where they had unequal votes.

3. Borda Tie-Breaking: Choose the candidate with the highest [lowest] Borda score. See [2].

4. Coombs Tie-Breaking: Choose the candidate with the fewest [most] last place votes.

It is possible that after applying one of these tie-breaking rules that the candidates would still be tied. Because of this, it is useful to distinguish between "weak ties" and "strong ties." A weak tie occurs when candidates have the same number of votes at a given stage. A strong tie occurs when candidates are still tied after applying a tie-breaking rule, such as one of the four listed above. A strong tie would be broken by lot.[1]

The ERS97 rules use forwards tie-breaking. The purpose of this paper is twofold. First, to show that backwards tie-breaking is a better solution and to suggest that the ERS97 rules be changed to use backwards tie-breaking instead. Second, to show that substage totals should not be used when breaking ties.

2   Backwards or Forwards Tie-Breaking

In breaking a tie, the ERS97 rules state that one must choose "the candidate who had the greatest vote [or fewest votes] at the first stage or at the earliest point in the count, after the transfer of a batch of papers, where they had unequal votes." This is forwards tie-breaking and is used when choosing a surplus to transfer (5.2.3), when choosing a candidate to eliminate (5.2.5), and when choosing winners (5.6.2).

The difference between backwards and forwards tie-breaking will be illustrated with the example in Table 6.1. In this example, we have to eliminate one candidate at stage 4 and there is a weak tie between candidates C and D. Thus, tie-breaking needs to be used to determine which candidate is to be eliminated. Under ERS97 rules, we break the tie by using forwards tie-breaking. To do this we first look to the counts at stage 1. We see that D has one more vote than C at stage 1. Thus, candidate C is eliminated.[2]

Another alternative is to use backwards tie-breaking. To do this, we look at the previous stage to break ties, and if necessary to preceding stages. Looking at the

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[1] Of course one could use another tie-breaking rule if the first tie-breaking rule results in a tie, but this will not be considered here. Borda and Coombs tie-breaking are just presented as available alternatives and will not be discussed further.

[2] If C and D had been tied at stage 1, then we would have looked to subsequent stages. If C and D had been tied at all stages, then we would have had a strong tie which would have been broken by lot.

preceding stage, we see that C is ahead of D at stage 3. Thus, D would be eliminated.

One problem with forwards tie-breaking is that it looks at the stages in an order that is not sequential. In order to determine the candidate to be eliminated at stage 4, we would look at the stages in the following order: 4 1 2 3. Intuitively, this is undesirable. It makes more sense to look at the stages in sequential order. Since one must look first to the current stage, there is only one sequential ordering: 4 3 2 1. This is what backwards tie-breaking would do.

A more important problem, is that forwards tie-breaking does not use the most relevant information to break the tie. The most relevant information to break a tie is the previous stage and not all the way back to the very first stage. By immediately looking to the first stage to break the tie, the ERS97 rules allow the tie-breaking to be influenced by candidates eliminated very early in the process and also by surpluses yet to be transferred. Instead, if we look to the previous stage to break a tie, candidates eliminated early on in the process will have no influence in breaking the tie. In addition, it allows for surpluses to be transferred which gives a more accurate picture of candidate strength.

In Table 6.1, candidate C has more support than candidate D at stage 3. At this point, the surplus of A has already been transferred and candidate F has already been eliminated. Thus, stage 3 is a better measure than is stage 1 as to which candidate should be eliminated at stage 4.

Other implementations of the single transferable vote use backwards tie-breaking instead of forwards tie-breaking: Cambridge, MA STV [3], rules advocated by the Center for Voting and Democracy [4], and rules advocated by the Proportional Representation Society of Australia [5].

3 Elimination of Winning Candidates

An incidental problem related to using forwards tie-breaking is that the ERS97 rules can sometimes eliminate a winning candidate. Consider an example where 31 voters elect one candidate with the following ballots:

4	voters vote	ABC
5	voters vote	BC
5	voters vote	CB
2	voters vote	DABC
4	voters vote	EABC
11	voters vote	F

Table 6.2 shows the results of the tally with ERS97 rules.

At stage 3 of the count, we need to eliminate one or more candidates and candidates B and C are tied with the fewest votes. According to rule 5.2.5(b), both B and C are to be eliminated. However, if instead the tie between B and C was broken by lot, then the other candidate would go on to win the election! In this scenario, suppose candidate C was eliminated by lot at stage three. Then B would be tied with A at stage 4, each with 10 votes. Forwards tie-breaking would be used to break the tie. Candidate A has the fewest votes at stage 1 and would then be eliminated. B would then receive all of A's votes and beat F 20 to 11 in the final stage.

Thus, the ERS97 rules are over-aggressive in eliminating candidates. This is a clear flaw in the ERS97 rules. This flaw arises from the interaction of rule 5.2.5(b) and forwards tie-breaking. This flaw could be fixed in two ways: (1) by changing rule 5.2.5(b), or (2) by using backwards tie-breaking instead of forwards tie-breaking. Since there are already other good reasons for using backwards tie-breaking, the obvious choice is (2).

If backwards tie-breaking were used instead, then both candidates B and C could properly be eliminated at stage 3. If just C were eliminated and B received all of C's votes, then there would again be a tie at stage 4. However, with backwards tie-breaking, B would necessarily have fewer votes than A at the previous stage and would immediately be eliminated.

Backwards tie-breaking would fix this flaw generally, and not just in this specific example. This flaw occurs under specific conditions:[3] (1) a candidate needs to be eliminated and two candidates are tied for last place, (2) the sum of the votes of these two candidates is equal to the candidate with the next fewest number of votes, and (3) after eliminating one of these candidates there would be a subsequent tie with this third candidate. Under these conditions rule 5.2.5(b) requires that the two candidates in last place be eliminated simultaneously. As described above, with forwards tie-breaking a winning candidate could be improperly eliminated. However, with backwards tie-breaking, both of these last-place candidates cannot win and can thus be properly eliminated. The two last-place candidates are guaranteed to lose the second tie because they necessarily

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[3]These conditions could be generalized to the case where more than two candidates are tied for last place.

have fewer votes at the previous stage (but they do not necessarily have fewer votes at the first stage).

4   Use of Substages to Break Ties

The word "substage" is not used anywhere in the ERS97 rules, but this terminology is used by people familiar with the rules. Substages can occur when transferring votes from eliminated candidates. Table 6.3 shows an example using ballots from the test T143 where 60 voters are electing two candidates. At stage 3, candidate F is being eliminated. Candidate F has ballots with transfer value 1.00 and ballots with transfer value 0.25 (from the surplus of A). These ballots will be transferred in two substages constituting two different batches. The first substage transfers ballots with value 1.00 and the second transfers ballots with value 0.25.

In stage 4 of this example, we need to eliminate a candidate and candidates C and D are tied for last place. Hence, we need to use forwards tie-breaking. With ERS97 rules, substages must be considered when doing forwards tie-breaking. Candidates C and D are also tied at stage 1 and stage 2, but candidate D is ahead of candidate C at the substage between stages 2 and 3. Thus, candidate C is eliminated.

The problem is that substages are not a good metric for breaking ties. In the example in Table 6.3, either candidate C or D must be eliminated at stage 4. Candidates C and D are tied at stages 4, 1, and 2. Candidate C is ahead at stage 3, but candidate C is eliminated anyway! The reason that C is eliminated is that D has more votes at an intermediary point where only some of candidate F's votes have been transferred. This intermediate point is well-defined but completely arbitrary in terms of fairness. There is no reason to make some of F's votes more important than others. Whether one candidate is ahead of another at this intermediary point is not relevant to which candidate should be eliminated. What is relevant, is what the counts are at each stage of the count, that is after a candidate has been completely eliminated.

5   Conclusions

The ERS97 rules should be changed so that backwards tie-breaking is used instead of forwards tie-breaking. In addition, substage totals should not be considered when breaking ties.

6   Acknowledgments

7   References

[1]   R A Newland and F S Britton. *How to conduct an election by the Single Transferable Vote*. ERS 3rd Edition. 1997. See http://www.electoral-reform.org.uk/votingsystems/stvrules.htm.

[2]   Earl Kitchener, Tie-Breaking in STV. *Voting matters*. Issue 11, April 2000.

[3]   Massachusetts General Laws, Chapter 54A, Section 9(k).

[4]   Choice Voting. The Center for Voting and Democracy. See http://www.fairvote.org/library/statutes/ choice_voting.htm

[5]   Rules of the Proportional Representation Society of Australia for conducting elections by the quota-preferential method. 2001. See http://www.prsa.org.au/rule1977.htm

		Surplus of A	Eliminate F	Eliminate E	Eliminate C
Stage	1	2	3	4	5
A	23	20.00	20.00	20.00	20.00
B	13	13.00	13.00	15.00	15.00
C	6	6.50	10.00	12.00	2.00
D	7	7.50	9.50	12.00	18.00
E	7	7.50	7.50	-	-
F	4	5.50	-	-	-
Non-Transferable	0	0.00	0.00	1.00	5.00

Table 6.1: Example tally with ERS97 rules where 60 voters are electing two candidates.

		Eliminate D	Eliminate E	Eliminate B & C
Stage	1	2	3	4
A	4	6.00	10.00	10.00
B	5	5.00	5.00	-
C	5	5.00	5.00	-
D	2	-	-	-
E	4	4.00	-	-
F	11	11.00	11.00	11.00
Non-Transferable	0	0.00	0.00	10.00

Table 6.2: Example where the ERS97 rules eliminate a winning candidate.
Thirty-one voters are electing one candidate. Candidate F is the winner.

		Surplus of A	Eliminate F		Eliminate E	Eliminate C
Stage	1	2	substage	3	4	5
A	23	20.00	20.00	20.00	20.00	20.00
B	13	13.00	13.00	13.00	15.00	15.00
C	7	7.50	8.50	10.00	12.00	2.00
D	7	7.50	9.50	9.50	12.00	18.00
E	6	6.50	6.50	6.50	-	-
F	4	5.50	1.50	-	-	-
Non-Transferable	0	0.00	1.00	1.00	1.00	5.00

Table 6.3: ERS97 rules with substage tie-breaking.
Sixty voters are electing two candidates.