Tie Breaking in STV

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1 Introduction

Given any specific counting rule, it is necessary to introduce some words to cover the situation in which a tie occurs. However, such ties are only a practical concern for small elections. For instance, it has been reported that a tie has never occurred with the rules used in the Irish Republic.

Probably the most common form of a tie is when the two smallest first preference votes are the same. Unless both candidates can be excluded, a choice must be made, although in very many cases, the candidates elected will be the same.

This note proposes that when a computer is used to undertake a count, all the possible choices should be examined and that the result is produced by computing the probability of election of the candidates.

2 Ties in practice

It is clear that the propensity to produce a tie will depend largely on the number of votes. However, some estimate can be obtained from a collection of election data that has recently been revised [1]. The data base consists of over 700 'elections', but for this paper we exclude artificial test cases. The figures obtained from the other cases, which are like real elections, with three counting rules ([4, 2, 7]) are as in the table on page 4.

Hence, although with the Church of England rules, only 59 out of 299 involved a tie-break, the average number of tie-breaks in those 59 was actually 9.9. The average number of votes in those 59 cases was 102, while the average for the remaining 240 cases was 12,900. It is important to note that Meek only has ties on an exclusion of a candidate, while the hand-counting rules also have ties on the choice of the candidate whose surplus is to be transferred.

For reasons not relevant to this note, the number of cases run with each rule is different. (Larger cases have only been run with Meek.) It is clear that a small number of votes increases the risk of a tie. Also, given that a tie occurs, the Meek algorithm has only half the risk of a subsequent tie arising, almost certainly due to the higher precision of the calculation.

3 The special case of ties with the Meek algorithm

Brian Meek's original proposal rests upon the solution of certain algebraic equations. The algorithm given in [7] provides an iterative solution of those equations. The mathematical nature of the equations implies that there is substantial freedom in handling exclusions, since, once a candidate is excluded, it is as if the candidate had never entered the contest. Hence it is not necessary for two implementations of Meek to handle exclusions in the same way — the same candidates will be elected. (In contrast, the hand counting rules need to be specific on exclusions since it affects the result; ERS97 insists on as many as allowable, while CofE insists on only one at a time.)

As an example, David Hill's implementation of Meek in comparison with my own has revealed differences. We both exclude together all those candidates having no first preferences. David Hill also excludes the next-lowest candidate also (assuming it is safe to do so), while I do not. I will exclude more than one candidate at a time when it is safe to do so, while David Hill sticks to one at a time. Hence both our implementations report a random choice has been made when it is certainly possible to avoid this. Such reporting is undesirable since it might give the impression that those elected have been chosen at random, when this is not the case. Both of us have introduced a tie-breaking rule similar to that in many hand-counting rules based upon the votes in previous stages (but in opposition to that advocated in [5]).

Two other aspects are relevant to the Meek algorithm. The cases reported in [6] indicate that an implementation can report a tie even though in mathematical terms, one candidate is ahead (but by too small an amount to be computed). This situation is not thought to arise in practice. Perhaps somewhat more disturbing is that an algebraic tie can be computed differently, giving one candidate ahead of another. Two implementations of Meek with such a case can even break the actual tie by rounding in different directions. However, since there is a real tie, breaking it by the rounding in the implementation, is not so bad.

4 Results of the proposed method

The only practical method to implement this proposal is to modify software that already implements an existing counting rule. Since I have my own implementation of Meek, I have modified this to analyse all choices when a tie occurs.

The modification works by executing the algorithm once for every possible choice when the rules require a 'random' choice. For my version of Meek, I have provided an option to remove the first-difference rule so that when this rule would otherwise be invoked, a random choice is made[1].

As an example, consider a real (simple) election, R033, having four candidates (A1. . . A4) for one seat. At the first stage, A2 and A3 have the smallest number of votes: if A2 is excluded, then A1 is elected; if A3 is excluded, then there is a tie between A2 and A4 for the next exclusion. These two alternatives also result in A1 being elected. So the final result is:

Probability from 5 choices from 3 passes.

Candidate	*Excluded?*	*Probability*
A1	no	1
A2	yes	0
A3	yes	0
A4	yes	0

We now know that the election of A1 is not dependent upon the random choices made. The computation involved three election runs. The middle column indicates that the candidates A2, A3 and A4 were all selected in one of the runs for random exclusion.

A more complex example is given by R009, electing 2 from 14 candidates with 43 votes. Here, the final table reads:

Probability from 1364 choices from 264 passes.

Candidate	*Excluded?*	*Probability*
A1	no	1/ 4
A2	yes	0
A3	yes	0
A4	yes	0
A5	no	0
A6	yes	0
A7	yes	0
A8	yes	0
A9	no	1
A10	yes	0
A11	no	3/ 4
A12	yes	0
A13	yes	0
A14	no	0

Here we see that only the candidates A1, A5, A9, A11 and A14 were never subject to random exclusion. Nevertheless, A5 and A14 were never elected.

However, the above result was using the variant of Meek without the first-difference rule. If the first-difference rule had been applied, then A1 would not have been elected in any circumstances. Note that in this case, a large number of passes had to be made due to many of the stages resulting in a tie. Hence this technique is only really possible due to the speed of modern computers.

Given the above election, then there are two possible uses of the outcome: firstly to elect the most probably candidates (A9 and A11), or secondly, to randomly select between A1 and A11 according to the specified possibilities. Since in this paper we are attempting to reduce the random element, we choose the first option.

From the database, 55 cases were selected which correspond reasonably closely to real elections. The results are in the table on page 5. The entry 'Random' gives the number of random choices made with the New Zealand version of Meek which has the first-difference rule. The last three entries are from running the new program. The 'Probs.' column includes the probabilities of election of those candidates who are involved in ties and have nonzero probability of election.

[1] The first-difference rule is a method of breaking a tie by examining the votes in all previous stages, starting at the first stage and selecting the one which has the fewest votes at the first stage at which there is a difference. Of course, if the earlier stages give no difference, then a random method must be used to break the tie.

The three examples with approximate results from the new program took too long to run to completion. Here, the tabulated results are based upon the first few thousand cases executed. The majority ran very quickly and only those with 10,000 or more passes took longer than a minute or two. The case R038 was exceptional in having probabilities of 29/168, 11/35, 29/60, 431/840, 431/1680, 437/1680 and 1 (and none were repeated).

If one was only concerned with the Meek algorithm, then the program could probably be made substantially faster since the ties only arise with an exclusion and Meek is indifferent to the order of the exclusions in the sense that excluding A then B is the same as excluding B then A; this situation will typically be the case when A and B tie on the fewest number of votes. The approach here is a general one that could be applied to any counting rule. It also seemed easier to program the general method presented here.

From the 49 cases which were run to completion, all but 7 reported than the random choice had no effect upon the result.

Election R102 is typical of the situation in which a large number of random choices are made. In fact, 28 exclusions are made before an election. This implies that for all these initial stages, the votes are integers. Given the small size of the election, ties are very common. Unfortunately, this implies that the number of choices is too large to compute them all. However, experimenting with removing those candidates who are excluded early, gives the result shown in the last column.

Followers of the Eurovision Song Contest might like to know that although the official scoring system gave a tie in 1991 between Sweden and France, with Sweden being judged the winner on the basis of having more second (preference) votes, this system gives Sweden a probability of election of 71/288 and hence France the clear winner with a probability of 217/288. According to this system, the UK would have won in 1992 with a probability of 5/6, while the official result declared Ireland as the winner which had a probability of only 1/12.

5 Conclusions

It seems that the provision of this program raises more problems than it solves. If one is prepared to ignore the 14% of cases which question the validity of the random choice, then one can continue the current practice with a clear conscience. On the other hand, when a random choice was made in a real election, it would surely be welcome to show that the result was not in question. However, using this program for that purpose might not give a clear answer when only a fraction of all the possibilities could be executed in a reasonable time (as with the three cases in the table). Of course, in those cases, numerous random choices could be tried, but the object here is to avoid such arbitrariness.

When a candidate has been subject to a random exclusion in an election, he/she could naturally feel aggrieved. One solution to that would be to undertake a re-count without randomly excluding that candidate. If this were undertaken by computer, the number of recounts would be less than the number of candidates and hence very much less than all possibilities which are considered above.

Currently, almost all STV counting rules introduce some rules, like the first-difference rule ([2, 4]) or Borda scores [3], to reduce the need for a random choice to be made. An alternative would be to simplify the counting rules by omitting these provisions, but to use a program like the one presented here to produce a result which is very likely to have no random element.

6 References

[1] B. A. Wichmann. A Guide to an STV data base. July 2004. (Available from the author.)

[2] R. A. Newland and F. S. Britton. *How to conduct an election by the Single Transferable Vote*. ERS 3rd Edition. 1997. See `http://www.electoral-reform.org.uk/votingsystems/stvrules.htm`

[3] Earl Kitchener, Tie-Breaking in STV. *Voting matters*. Issue 11, April 2000.

[4] GS1327: General Synod, Single Transferable Vote regulations 1990 and 1998. (Obtainable from Church House Bookshop, Great Smith Street, London SW1P 3BN.)

[5] J. C. O'Neill: Tie-Breaking with the Single Transferable Vote. *Voting matters* Issue 18. pp 14-17. 2004.

[6] B. A. Wichmann. The computational accuracy using the Meek algorithm. *Voting matters* Issue 12. pp6-7. November 2000.

[7] I. D. Hill, B. A. Wichmann and D. R. Woodall. Algorithm 123 — Single Transferable Vote by Meek's method. *Computer Journal*. Vol 30, pp277-281, 1987.

Rule	*Ties*	*Ties per case*	*Average votes with ties*	*Average votes without ties*
CofE	59 from 299	9.9	102	12900
ERS97	55 from 154	7.1	81	2438
Meek	62 from 587	3.3	12692	44180

Table 1.1: Ties with different election rules

ID	Votes	Candidates	Seats	Random	Choices	Passes	Probs.
M002	131	20	5	1	2	2	1
M112	692	25	6	1	2	2	1
R009	43	14	2	4	1364	264	1/4, 3/4, 1
R012	79	17	2	4	256	48	1
R015	83	19	3	6	32640	3840	1
R017	76	20	2	5	64776	7200	1
R018	104	26	2	11	—	$\approx 6 \times 10^{6}$	1?
R019	73	17	2	5	3876	672	1
R020	77	21	2	5	42184	4572	5/24, 19/24, 1
R027	44	11	2	4	114	30	1
R028	91	29	2	8	—	$\approx 5 \times 10^{6}$	1?
R033	115	4	1	1	5	3	1
R038	9	18	3	3	387	115	*see text*
R040	176	17	5	1	2	2	1
R097	45	17	1	6	283742	31190	1
R100	1031	31	10	1	2	2	1
R102	247	49	10	15	—	$\approx 34 \times 10^{6}$	1/12, 1/4, 1/6, 2*5/6, 2*11/12, 6*1?
S002	16	16	1	1	8	4	2 of 1/2
S003	16	16	1	1	7	5	1
S004	20	20	1	2	12	6	1
S005	18	18	1	1	3	3	1
S006	20	20	1	3	60	18	1
S007	19	19	1	2	46	14	1
S008	19	19	1	3	106	31	1
S009	20	20	1	2	20	10	1
S010	22	22	1	3	448	106	1
S011	21	21	1	4	465	97	1
S012	22	22	1	1	2	2	1
S013	22	22	1	3	3888	624	1
S014	22	22	1	1	176	44	71/288, 217/288
S015	23	23	1	3	646	126	2 of 1/12, 5/6
S016	25	25	4	1	2	2	1
S022	25	25	1	4	1592	329	1
S023	23	23	1	3	288	60	1
S024	17	16	1	2	58	16	1
S025	18	18	1	4	480	96	2 of 1/2
S026	18	18	1	5	39703	6297	1
S027	13	19	1	2	30	12	1
S028	17	18	1	3	229	68	1
S029	18	18	1	2	16	7	1
S030	20	20	1	4	1368	288	1
S031	19	19	1	1	2	2	1
S032	16	19	1	2	16	7	1
S033	22	23	1	2	1132	206	1
S034	25	25	1	4	5774	1072	1
S035	25	25	1	6	70560	10080	1
S036	23	23	1	5	14400	2304	1
S037	24	24	1	2	16	7	1
S038	23	23	1	2	28	10	1
S039	26	26	2	5	17760	2880	1
S047	36	24	1	6	12144	1800	1
S048	24	24	1	6	161280	20160	1

Table 1.2: All results from exhaustive tie-breaking