Sequential STV — a further modification

I.D. Hill and Simon Gazeley
simon.gazeley@btinternet.com

1 Introduction

We had hoped that our earlier paper [1] would be the final version of the Sequential STV system, but we have found two examples since then that seem to call for further amendment.

The aim is to find a system that will be noticeably like ordinary STV but: (1) will correct unfairness, if any, to candidates excluded by the reject-the-lowest rule; (2) will automatically reduce to Condorcet's method rather than Alternative Vote when there is only a single seat.

It seeks to find a set of n candidates that observes Droop Proportionality [3], which we regard as an essential feature of any worthwhile voting system, and is preferred by the largest majority of voters to any other possible set of n. Tideman's CPO-STV [2] has similar objectives. The successful set will usually be such that any set of n+1 candidates, consisting of those n and 1 more, will result in the election of those n when an STV election is performed and in this case we refer to the successful set as a Condorcet winning set.

In a small election, or when n=1, it would be relatively easy and quick to do a complete analysis, as CPO-STV does. The challenge is to find a way that will work in a reasonable time in large elections, where such a complete analysis would be impracticable. We recognise that the meanings of 'a reasonable time' and 'impracticable' are open to dispute, and that what is practicable will change as computers continue to get faster. As Tideman and Richardson say "We are not yet at a point where computation cost can be ignored completely".

In cases where it is practicable to do a complete analysis of all sets of n+1, n+2, etc., it might be possible to find a solution that, in some sense, is preferable to that produced by this system that (after an initial stage) looks only at sets of n+1 and only at some of those. We think, however, that it would be hard to claim a severe injustice to any non-elected candidate after this system had been used, and it does keep things within manageable limits. It would be interesting to compare the performance of Sequential STV and CPO-STV, but this has not been done yet.

Of the two worrying examples, one showed that the system, as previously given, could fail to preserve Droop Proportionality, while the other showed that we were a little over-optimistic in claiming that, if the special procedure to deal with a Condorcet paradox had to be invoked, "most of the original candidates will be either excluded or certainties, [so] there is no need to fear an astronomical number of tests needing to be made". This second example was highly artificial and the optimism was probably justified for any real voting pattern that is at all likely to occur, but even artificial patterns ought not to cause trouble.

To cure the first of these troubles it is necessary, when the special procedure is used, to let it exclude just one candidate before restarting the main method, instead of continuing to use the special procedure. To cure the second, the special procedure has been much simplified, to calculate a value for each continuing candidate based upon Borda scores, and to exclude the one with the lowest score. We emphasise that in real elections, as distinct from specially devised test cases, Condorcet loops rarely occur and so the special procedure is rarely called into use.

Borda scores on their own, as an electoral method, we regard as a very poor option. Those elected are far too dependent upon whether or not other (non-winning) candidates are standing, and the method is much too open to tactical voting; but as a method of helping to sort out a Condorcet paradox, they can be useful. Where a paradox arises, we know that there cannot be a good result because, whoever is elected, it is possible to point to some other option that a majority of the voters preferred; so the best that can be done is to try for a not-too-bad result and, for this limited purpose, Borda scores can serve.

2 Revised version of Sequential STV

All STV counts mentioned are made by Meek's method. It would be possible to use a similar system with some other version of STV but, since many counts are to be made using the same data, to try it other than by computer would make little sense. If a computer is required in any case, Meek's method is to be preferred.

An initial STV count is made of all candidates for n seats, but instead of dividing into those elected and those not elected, it classifies those who would have been elected as probables, and puts the others into a queue, in the reverse order of their exclusion in that STV count, except that the runner-up is moved to last place as it is already known that an initial challenge by that candidate will not succeed. Having found the probables and the order of the queue, further rounds each consist of n+1 candidates, the n probables plus the head of the queue as challenger, for the n seats. Should a tie occur during these rounds, between a probable and a challenger, it is resolved by maintaining the current situation; that is to say, the challenger has not succeeded.

If the challenger is not successful, the probables are unchanged for the next round and the challenger moves to the end of the queue, but a successful challenger at once becomes a probable, while the beaten candidate loses probable status and is put to the end of the queue. The queue therefore changes its order as time goes on but its order always depends upon the votes.

This continues until either we get a complete run through the queue without any challenger succeeding, in which case we have a solution of the type that we are seeking, or we fall into a Condorcet-style loop.

A loop may have been found if a set that has been seen before recurs as the probables. If the queue is in the same order as before then a loop is certain and action is taken at once. If, however, a set recurs but the queue is in a different order, a second chance is given and the counting continues but, if the same set recurs yet again, a loop is assumed and action taken.

In either event the action is the same, to exclude all candidates who have never been a probable since the last restart (which means the start where no actual restart has occurred) and then to restart from the beginning except that the existing probables and queue are retained instead of making a new initial STV count.

If there is no candidate who can be so excluded, then a special procedure is used, in which each continuing candidate, other than any who has always been a probable since the last restart, is classified as 'at-risk'. Taking each continuing candidate, a Borda score is calculated, as the sum over all votes of the number of continuing candidates to whom the candidate in question is preferred, taking all unmentioned continuing candidates as equal in last place. A continuing candidate who is not mentioned in a particular vote is given, for that vote, the average score that would have been attained by all those unmentioned. In practice it can help to give 2 points instead of 1 for each candidate beaten, because all scores, including any averages required, are then whole numbers.

The at-risk candidate with the lowest score (or a random choice from those with equal lowest score) is then excluded and the main method restarted from the beginning, except that the existing probables and queue order are retained instead of making the initial STV count. If the newly excluded candidate was one of the queue, he or she is merely removed from the queue, but if the candidate was a probable, the candidate at the head of the queue is reclassified as a probable and removed from the queue. Then a restart is made from the beginning except that the existing probables and queue are retained instead of making a new initial STV count.

3 Proof of Droop Proportionality compliance

The 'Droop proportionality criterion' says that if, for some whole numbers k and m (where k is greater than 0 and m is greater than or equal to k), more than k Droop quotas of voters put the same m candidates (not necessarily in the same order) as their top m preferences, then at least k of those m candidates will be elected.

We know that a normal STV count is Droop Proportionality compliant so, in Sequential STV, for k and m defined as above, at least k of the m will be probables at the first count. If on a later count a challenger takes over as a probable then, because that was also the result of an STV count, there will still be at least k of the m among the probables, even if the replaced candidate was one of the m. This ensures compliance if no paradox is found.

If a paradox is found, at least k of the m will have been probables at some time since the last restart, so excluding all who have not been probables must leave at least k. If the special procedure, using Borda scores, is required, then if only k exist, k will have always been probables since the last restart, and so are not at risk of exclusion, but if there are more than k, the exclusion of just one of them must leave at least k. This ensures compliance where a paradox is found.

4 Examples

Example 1

This is the example that showed the old version of Sequential STV to fail on Droop Proportionality. With 9 candidates for 3 seats, votes are

10	ABCDEFGH	10	BCDAFGHI
10	CDABGHIE	11	DABCHIEF
19	EFGHIDAB	19	FGHIEBCD
1	GHIEFCDA		

41 votes (more than 2 quotas) have put ABCD, in some order, as their first choices so, to satisfy Droop Proportionality, at least 2 of them must be elected. The old version elected DEF but the new version elects ADE.

Example 2

This is the example that showed the old version of Sequential STV not always to finish within a reasonable time. With 40 candidates for 9 seats, votes are

69	ABCDE	94	BCAED	98	CBAED
14	DEBAC	60	ECBDA	64	FGJHI
43	GIFJH	42	HJIGF	97	IHGJF
33	JIHGF	32	KLMNO	44	LMNOK
56	MNOKL	76	NOKLM	90	OKLMN
18	PQRST	91	QRSTP	69	RSTPQ
21	STPQR	76	TPQRS	36	UVWXY
78	VWXYU	99	WXYUV	29	XYUVW
4	YUVWX	64	abcde	35	bcdea
69	cdeab	98	deabc	16	eabcd
40	fghij	44	ghijf	79	hijfg
42	ijfgh	68	jfghi	13	kmnop
64	mnopk	83	nopkm	30	opkmn
33	ponmk				

This new version of Sequential STV terminates after 835 STV counts, whereas the old version would, we estimate, have required over 177,000 counts. We emphasise again that the voting pattern is highly artificial — in a real election, with 40 candidates for 9 seats, more than 60 counts would be very unusual.

Example 3: "Woodall's torpedo"

With 6 candidates for 2 seats, votes are

11	AC	9	ADEF	10	BC
9	BDEF	10	CA	10	CB
10	EFDA	11	FDEB		

Sequential STV elects CD even though AB form the unique Condorcet winning set. Examining why this happens, it is found that A and B are always elected by STV from any set of 3 in which they are both present, but neither A nor B is ever elected if one of them is there but not the other. Meanwhile C is always elected if present in a set of 3 except for the one set ABC. D, E and F form a Condorcet loop. CD, CE or CF would be a second Condorcet winning set if the other two of D, E and F were withdrawn.

Such a strange voting pattern is unlikely to arise in practice. It shows that Sequential STV cannot be guaranteed to find a Condorcet winning set even where one exists but it does not shake our belief that Sequential STV is a good system; it would be hard to deny that C is a worthier winner than either A or B in this example.

5 Acknowledgements

We thank Douglas Woodall for devising example 3, and the referee for useful comments on earlier versions of this paper.

6 References

[1] I.D. Hill and Simon Gazeley, Sequential STV — a new version. *Voting matters*, 15, 13–16. 2002.

[2] T. Nicolaus Tideman and Daniel Richardson, Better voting methods through technology: the refinement-manageability trade-off in the single transferable vote. *Public Choice*, 103, 13–34. 2000.

[3] D.R. Woodall, Properties of preferential election rules. *Voting matters*, 3, 8–15. 1994.