Review – Voting Theory for Democracy

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Abstract

Mathematica is the most important information visualization software. It is proprietary software developed by Wolfram Research. In the book *Voting Theory for Democracy* (323 pages) [1], Thomas Colignatus describes and motivates his voting software add-on (*Economics Pack*) for *Mathematica*. This book is also intended to be a primer in voting theory.

1 Introduction

Section 1 (19 pages) explains Condorcet's paradox and Arrow's theorem. Unfortunately, there are no formal definitions for *preferences*, *orderings*, *single-seat elections*, etc.

I disagree with the author's interpretation of Arrow's theorem. For example, the author writes (page 31): "It is sometimes thought that all problems in voting are caused by Arrow's theorem. This however is a misunderstanding. The problems in voting are not caused by Arrow's theorem but by the possibility of cheating." However, it has been shown by Gibbard [2] and Satterthwaite [3] that it is a direct consequence of Arrow's theorem that all preferential single-seat election methods, that are Paretian and non-dictatorial, are vulnerable to "cheating" (strategic voting).

Furthermore, the author argues that Arrow's theorem is unreasonable because candidates always have to cast an eye not only on all the other candidates, but also on all those politicians who might declare their candidacy at a later time. Therefore, for the concrete campaign strategies of candidate *a*, it doesn't matter whether politician *b* has already announced his candidacy or might do this later. In the words of the author (page 30): "In voting, the relative positions of two candidates might depend upon the budget [that is, the set] of available candidates." However, in my opinion, the author only moves the problem of *irrelevant alternatives* from asking, whether *b* is a *declared* candidate, to asking, whether *b* is an *available* candidate. When *b*, who didn't announce his candidacy, dies and when (in reaction to this change of the pool of available candidates) the other candidates change their positions and when this leads to a change of the final winner, this is still an undesirable event.

Section 2 (2 pages) explains the installation process for the *Economics Pack*. **Section 3** (26 pages) explains the possible formats for the input for the programs to calculate the winners of the different single-seat election methods.

Sections 4 and 5 (72 pages in total) explain the possible visualizations of the input (e.g. pairwise digraphs, Black diagrams, Saari diagrams). Furthermore, they explain all the single-seat election methods whose winners can be calculated with the *Economics Pack*: plurality voting, top-two runoff, Borda, Nanson, and the "Borda Fixed Point" method.

The author's use of some terms differs significantly from their use in the scientific literature. This leads to misunderstandings when, for example, the author concludes that "plurality voting can violate Pareto optimality" (page 70). The author also criticizes the Borda count for violating "Pareto optimality".

Section 6 (24 pages) discusses strategic voting and the no-show paradox. The author presents some examples. But unfortunately, he doesn't present general results.

Section 7 (42 pages) describes the Elo rating system (a method for ranking the relative skill levels of players in head-to-head games; e.g. chess) and the Rasch model (a method for ranking students according to their performance in psychological tests). There is no analysis of these schemes. Without any analysis, the author reaches the conclusion that these schemes are also suitable for public elections.

Section 8 (31 pages) tries to estimate the cardinal utilities of the voters. **Sections 9 and 10** (71 pages in total) discuss Arrow's theorem. At one point, the author "solves" Arrow's theorem by rejecting *independence of irrelevant alternatives* as unreasonable. At another point, the author "solves" Arrow's theorem by keeping the election method undefined in the case of circular ties.

2 The "Borda Fixed Point" Method:

A serious problem of this book is that the author spends too much time introducing his own pet method: the "Borda Fixed Point" (BFP) method. This method has neither been published nor adopted somewhere. Even this book doesn't contain a proper analysis of this method. So why should we be interested in software to calculate the winner of the BFP method?

The *Borda complement* BC[*x*] of candidate *x* is that candidate who would be the Borda winner if candidate *x* didn't run. Candidate *x* is a *Borda Fixed Point candidate* if he pairwise beats BC[*x*]. The *Borda Fixed Point winner* is the winner of a Borda count among all Borda Fixed Point candidates.

The basic idea of the BFP method is that, when candidate *x* is added to the pool of candidates, then candidate *x* should be able to win only by being a better candidate and not simply by the fact that, by his addition to the pool of candidates, this pool is perturbed in such a manner that candidate *x* happens to be chosen by the used election method. The author calls this the "proposal-versus-alternative approach". A new candidate should be able to win only if he is an "improvement" from the original winner (i.e. only if he pairwise beats the original winner).

The author claims that the BFP method satisfies the proposal-versus-alternative condition. But the following examples show that it doesn't.

Example 1

51 *abcde*
49 *cdeba*

We get BC[*a*] = *c*. *a* pairwise beats *c*. Therefore, *a* is a BFP candidate.
We get BC[*b*] = *c*. *b* pairwise beats *c*. Therefore, *b* is a BFP candidate.
We get BC[*c*] = *d*. *c* pairwise beats *d*. Therefore, *c* is a BFP candidate.
We get BC[d] = *c*. *d* doesn't pairwise beat *c*. Therefore, *d* is not a BFP candidate.
We get BC[*e*] = *c*. *e* doesn't pairwise beat *c*. Therefore, *e* is not a BFP candidate.
Now, the Borda count is applied to the BFP candidates:

51 *abc*
49 *cba*

The winner of this Borda count is *a*. Therefore, the BFP winner is *a*.

Example 2

51 *afbcde*
49 *cdefba*

We get *BC*[*a*] = *c*. *a* pairwise beats *c*. Therefore, *a* is a BFP candidate.
We get BC[*b*] = *c*. *b* pairwise beats *c*. Therefore, *b* is a BFP candidate.
We get BC[*c*] = *f*. *c* doesn't pairwise beat *f*. Therefore, *c* is not a BFP candidate.
We get BC[*d*] = *f*. *d* doesn't pairwise beat *f*. Therefore, *d* is not a BFP candidate.
We get BC[*e*] = *f*. *e* doesn't pairwise beat *f*. Therefore, *e* is not a BFP candidate.
We get BC[*f*] = *c*. *f* pairwise beats *c*. Therefore, *f* is a BFP candidate.
Now, the Borda count is applied to the BFP candidates:

51 *afb*
49 *fba*

The winner of this Borda count is *f*. Therefore, the BFP winner is *f*.

Thus the newly added candidate *f* changes the BFP winner from candidate *a* to candidate *f* without pairwise beating candidate *a*.
The author claims that the BFP method satisfies the majority criterion. But example #2 shows that it doesn't.
Furthermore, there are other single-seat election methods (e.g. the Kemeny-Young

method [4] and Tideman's ranked pairs method [5]) where a newly added candidate x can win only if he pairwise beats that candidate who would be elected if candidate x didn't run. So the problem that Colignatus addresses has already been solved in the scientific literature.

Furthermore, I don't consider the proposal-versus-alternative condition important because, when the number of candidates is increased from N to $N+1$, then the newly added candidate is always chosen in $1/(N+1)$ of all profiles. Therefore, instead of trying to minimize the number of profiles where the newly added candidate wins, one should rather try to minimize the number of profiles where the newly added candidate x changes the winner from candidate y to some other candidate $z \notin \{x, y\}$.

3 Summary:

This book is not suitable as a primer in voting theory because (1) this book contains too many errors, (2) the author spends too much time on his own pet method, and (3) in too many cases the author's use of terms differs too much from their use in the scientific literature. The author addresses too many topics; but he doesn't address them properly and thoroughly. The main problem of this book is the lack of formal definitions. (For example: The fact, that the author can "solve" Arrow's theorem by keeping the election method undefined in some cases, is only possible because he didn't give a formal definition for election methods.) The author's criticism of Arrow's theorem (which covers about one fourth of this book) is just mumbo-jumbo.

4 References

[1] Thomas Colignatus, "*Voting Theory for Democracy*", third edition, 2011, ISBN 9789080477483, http://www.dataweb.nl/~cool/Papers/VTFD/VotingTheoryForDemocracy.pdf.

[2] Allan Gibbard, "Manipulation of Voting Schemes: A General Result", *Econometrica*, volume 41, number 4, pages 587–601, 1973.

[3] Mark A. Satterthwaite, "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions", *Journal of Economic Theory*, volume 10, issue 2, pages 187–217, 1975.

[4] H. Peyton Young, "Condorcet's Theory of Voting", *American Political Science Review*, volume 82, number 4, pages 1231–1244, 1988.

[5] T. Nicolaus Tideman, "Independence of Clones as a Criterion for Voting Rules", *Social Choice and Welfare*, volume 4, number 3, pages 185–206, 1987.