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Voting matters

To advance the understanding of preferential voting systems

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For further information on the Trust, please contact:

The Secretary,
McDougall Trust,
6 Chancel Street,
London SE1 0UX, UK.
Telephone: +44 (0)20 7620 1080
Facsimile: +44 (0)20 7928 1528
Email: admin@mcdougall.org.uk
Web: www.mcdougall.org.uk

For further information on this publication, please contact Nicolaus Tideman, the Editor, at Economics Department, Virginia Tech, Blacksburg VA 24061, USA, or by email at ntideman@vt.edu.

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Editorial

There are five items in this issue:

- The first paper, by James Green-Armytage, describes four voting procedures for electing a single candidate from ranked preferences of voters. The four procedures differ very slightly from one another, and all are notable for electing the Condorcet winner when there is one and for strongly limiting the opportunity to benefit from strategic voting.
- In the second paper, David Hill explains the virtues, in terms of representativeness and the minimization of wasted votes, of having voters rank parties and using transfers in the vote counting, if party lists are to be used to elect a set of representatives.
- In the third paper, Peter Emerson makes a case for using the matrix vote to elect a collection of leaders and, with the same ballot, to name a person to each leadership position.
- The fourth paper is a discussion by Svante Janson of the virtues of using an exact Droop quota rather than a rounded Droop quota.
- The fifth and final item is Markus Schulze's review of *Voting Theory for Democracy*, by Thomas Colignatus.

Four Condorcet-Hare Hybrid Methods for Single-Winner Elections

James Green-Armytage
j.armytage@gmail.com

Abstract

This paper examines four single-winner election methods, denoted here as Woodall, Benham, Smith-AV, and Tideman, that all make use of both Condorcet's pairwise comparison principle and Hare's elimination and reallocation principle used in the alternative vote. These methods have many significant properties in common, including Smith efficiency and relatively strong resistance to strategic manipulation, though they differ with regard to the minor properties of 'Smith-IIA' and 'mono-add-plump'.

1 Introduction

The concept of majority rule is trickier than most people realize. When there are only two candidates in an election, then its meaning is quite clear: it tells us that the candidate with the most votes is elected. However, when there are more than two candidates, and no single candidate is the first choice of a majority, the meaning is no longer obvious.

The Condorcet principle[1] offers a plausible guideline for the meaning of majority rule in multi-candidate elections: if voters rank candidates in order of preference, and these rankings indicate that there is a candidate who would win a majority of votes in a one-on-one race against any other candidate on the ballot (a Condorcet winner), then we may interpret 'majority rule' as requiring his election.

The weakness of this guideline is that it does not specify what majority rule requires when there is no Condorcet winner. For these situations, the Smith set provides a useful generalization of the Condorcet winner concept. The Smith set is the smallest set S such that any candidate in S would win a one-on-one race against any candidate not in S. Thus the Smith principle, which requires voting rules to select winning candidates from the Smith set, is an extension of the Condorcet principle that is applicable to all election outcomes.[2] For example, suppose that A is preferred by a majority to B, B is preferred by a majority to C, C is preferred by a majority to A, and all three of these candidates are preferred by majorities to D. In this case, electing A, B, or C is consistent with the majority rule guideline provided by the Smith principle, but electing D is not.

Several election methods have been proposed that satisfy the Smith principle. Among them are ranked pairs,[3] beatpath,[4] river,[5] Kemeny,[6] Nanson,[7] and Copeland.[8] However, the four methods on which this paper focuses possess another property, in addition to Smith efficiency, that makes them particularly interesting: they appear to be unusually resistant to strategic manipulation. Therefore, if a society wishes to choose among multiple options by majority rule given one balloting, and if it wishes to minimize the probability that

[1] Condorcet (1785) defines this principle.

[2] Smith (1973) refers to his idea as a generalization of Condorcet consistency.
[3] Defined in Tideman (1987).
[4] Defined in Schulze (2003).
[5] Defined in Heitzig (2004).
[6] Defined in Kemeny (1959).
[7] See Tideman (2006), page 201–203.
[8] Defined in Copeland (1951).

voters will have an incentive to behave strategically, these methods are worthy of strong consideration.

These four methods also share the characteristic of employing the 'Hare principle', that is, the principle of eliminating the candidate with the fewest first-choice votes and reallocating those votes to other candidates.[9]

I will use the names Woodall, Benham, Smith-AV, and Tideman to refer to these rules, as they do not have standard names. They are deeply similar to one another and will choose the same winner in the vast majority of cases, but they are not identical. The purpose of this paper is to provide a solid understanding of how these methods work, how they differ from one another, and how they compare to other single-winner methods.

2 Preliminary Definitions

Assume that there are C candidates and V voters. Let τ be a tiebreaking vector that gives a unique score $\tau_c \in (0,1)$ to each candidate $c = 1, \dots, C$; τ can be random, predetermined, or determined by a tie-breaking ranking of candidates.[10] Let E be a vector of candidate eliminations, such that E_c is initially set to zero for each candidate $c = 1, \dots, C$. Let w denote the winning candidate. Let U_{vc} be the utility of voter v for candidate c. Let R_{vc} be the ranking that voter v gives to candidate c (such that lower-numbered rankings are better). All voting methods described in this paper, with the exception of approval voting and range voting, begin with the voters ranking the candidates in order of preference.

Pairwise comparison: An imaginary head-to-head contest between two candidates, in which each voter is assumed to vote for the candidate whom he gives a better ranking to. Formally, let $P_{xy} = \sum_{v=1}^{V} 1\{R_{vx} < R_{vy}\}$ be the number of voters who rank candidate x ahead of candidate y. If $P_{xy} > P_{yx}$, then x pairwise-beats y.

Condorcet winner: A candidate who wins all of his pairwise comparisons. Formally, x is a Condorcet winner if and only if $P_{xy} > P_{yx}, \forall y \neq x$.

Condorcet method: Any single-winner voting rule that always elects the Condorcet winner when one exists.

Majority rule cycle: A situation in which each of the candidates suffers at least one pairwise defeat, so that there is no Condorcet winner. Formally, $\forall x, \exists y: P_{yx} > P_{xy}$.

The Alternative Vote (AV):[11] The candidate with the fewest first choice votes (ballots ranking the candidate above all others in the race) is eliminated. The process is repeated until only one candidate remains.

Formally, in each round $r = 1, \dots, C-1$, we perform the following operations:

$$I_{vc} = 1\{[E_c = 0] \wedge [R_{vc} < R_{vc'}, \forall c' : (E_{c'} = 0 \wedge c' \neq c)]\}, \forall v, c.$$

$$F_c = \sum_{v=1}^{V} I_{vc} + \tau_c + E_c, \forall c.$$

$$z = \operatorname{argmin}(F).\ E_z = \infty.\ \Omega_z = r.$$

After round $C-1$, $w = \operatorname{argmin}(E)$, and $\Omega_w = C$.

Here, I is a V by C matrix indicating individual voters' top choices. F is a length-C vector of the candidates' first choice vote totals, which incorporates the unique fractional values in the tiebreaking vector τ in order to ensure that there will not be a tie for plurality loser. Infinity can be added to the F values of eliminated candidates to prevent them from being identified as the plurality loser in subsequent rounds. The vector Ω gives an 'elimination score' for each candidate, which will be used by the Woodall method.

Smith set:[12] Or, the 'minimal dominant set'. The smallest set of candidates such that every

[9] Thomas Hare offered the first voting procedure that included the iterative transfer of votes from plurality losers to candidates ranked next on ballots. See Hoag and Hallett (1926, 162–95). The first person to apply the 'Hare principle' to the election of a single candidate was Robert Ware, in 1871. See Reilly (2001, 33–34).

[10] See Zavist and Tideman (1989).

[11] Also known as instant runoff voting (IRV) and as the Hare method, the alternative vote (AV) is the application of proportional representation by the single transferable vote (STV) to the case of electing one candidate.

[12] This is so named because of Smith (1973). Schwartz (1986) refers to the Smith set as the

candidate inside the set is preferred by some majority of the voters to every candidate outside the set. When there is a Condorcet winner, it is the only member of the Smith set. Formally, the Smith set is the set of candidates S such that these conditions hold:

$\forall x \in S, \forall y \notin S, P_{xy} > P_{yx}$.

$\nexists S' \subsetneq S: (\forall x \in S', \forall y \notin S', P_{xy} > P_{yx})$.

3 Method Definitions

Woodall method:[13] Score candidates according to their elimination scores, and choose the Smith set candidate with best score.

That is, define each candidate's elimination score as the round in which he is eliminated by AV. (The AV winner is not eliminated, so we set his score to C.) If the Smith set has only one member, then this is the Woodall winner; otherwise, the winner is the candidate from inside the Smith set who has the best elimination score.

Formally, we begin with the definitions of the AV method and Smith set as given above. Then, $\Upsilon_c = 1\{c \in S\} \cdot \Omega_c, \forall c$, and $w = \text{argmax}(\Upsilon)$.

Benham method:[14] Eliminate the plurality loser until there is a Condorcet winner.

That is, if there is a Condorcet winner, he is also the Woodall winner. Otherwise, the method eliminates the candidate with the fewest first-choice votes, and checks to see whether there is a candidate who beats all other non-eliminated candidates pairwise. This process repeats until there is such a candidate, who is then declared the winner.

Formally, in each round we determine whether

$\exists x: \left[\left(P_{xy} > P_{yx}, \forall y: E_y = 0\right) \wedge (E_x = 0)\right]$.

If so, then $w = x$, and the process stops. Otherwise, we perform these calculations:

$I_{vc} = 1\{[E_c = 0] \wedge [R_{vc} < R_{vc'}, \forall c'$:

$(E_{c'} = 0 \wedge c' \neq c)]\}, \forall v, c$.

$F_c = \sum_{v=1}^{V} I_{vc} + \tau_c + E_c, \forall c$.

$z = \text{argmin}(F)$. $E_z = \infty$.

Then, we proceed to the next round.

Smith-AV method:[15] Eliminate candidates not in the Smith set, and then conduct an AV tally among remaining candidates.

Tideman method:[16] Alternate between eliminating all candidates outside the Smith set, and eliminating the plurality loser, until one candidate remains.

That is, as in Smith-AV, we begin by eliminating all candidates outside the Smith set. If this leaves only one candidate (a Condorcet winner), then he is elected. Otherwise, we eliminate the candidate with the fewest first choice votes. Then, we recalculate the Smith set, and eliminate any candidates who were in it before but are no longer in it as a result of the plurality loser elimination. These two steps repeat until only one candidate (the winner) remains.

Formally, in stage 1, we define or re-define S according to the following conditions:

$\forall x \in S, \forall y$:
$\left(y \notin S \wedge E_y = 0\right), P_{xy} > P_{yx}. \forall x \in S, E_x = 0$.

$\nexists S' \subsetneq S$:
$[\forall x \in S', \forall y: (y \notin S' \wedge E_y = 0), P_{xy} > P_{yx}]$.

Then, we make the following adjustment to the E vector: $c \notin S \rightarrow E_c = \infty$.

In stage 2, we perform the following calculations:

GETCHA set, and also defines another set called the GOCHA set, which is now also known as the Schwartz set. The Schwartz set is the union of minimal undominated sets, where an undominated set is a set such that no member of the set is pairwise-defeated by a non-member. (This is equivalent to the Smith set in the absence of pairwise ties.) Though the methods defined in this paper are based on the Smith set, each has a potential Schwartz-set counterpart.

[13] Woodall (2003) defines this method (among many, many others), and refers to it as CNTT, AV, for 'Condorcet (net) top tier, alternative vote'.

[14] I'm not aware of any academic papers that define this method, but it was suggested to me by Chris Benham.

[15] Woodall (1997) lists this method under the heading 'naïve rules'. I refer to it as Smith-AV because it seems like the most obvious combination of the Smith set and AV.

[16] Tideman (2006) defines this method on page 232 and refers to it as alternative Smith.

$I_{vc} = 1\{[E_c = 0] \wedge$
$[R_{vc} < R_{vc'}, \forall c' : (E'_c = 0 \wedge c' \neq c)]\}, \forall v, c.$

$F_c = \sum_{v=1}^{V} I_{vc} + \tau_c + E_c, \forall c.$

$z = \mathrm{argmin}(F).\ E_z = \infty.$

Stages 1 and 2 alternate until S has only one member, i.e. $S = \{w\}$.

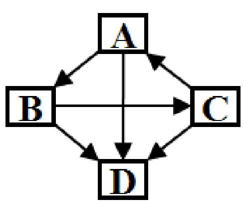
4 Examples

Examples 1 and 2 demonstrate how the four methods work, and prove that none of them are equivalent to any of the others. To help illustrate each calculation, I present the pairwise matrix, P, and a corresponding tournament diagram that uses arrows to represent pairwise defeats. I also present round-by-round tallies for the different methods, which show how many first choice votes each candidate holds at each stage of the count, along with the transfers of first choice votes from eliminated candidates.

Example 1: Woodall and Benham differ from Smith-AV and Tideman

6	DABC
5	BCAD
4	CABD

P	A	B	C	D
A		10	6	9
B	5		11	9
C	9	4		9
D	6	6	6	



	r1		r2	
A	0	X	-	
B	5		5	✔
C	4		4	X
D	6		6	X

Benham tally

	r1		r2		r3	
A	0	X	-		-	
B	5		5	+4	9	✔
C	4		4	X	-	
D	6		6		6	X

AV tally

	r1		r2		r3	
A	0	+6	6	+4	10	✔
B	5		5		5	X
C	4		4	X	-	
D	6	X	-		-	

Smith-AV or Tideman tally

Woodall: In an AV tally, A is eliminated first, followed by C and then D, leaving B as the winner. The Smith set is {A,B,C} Therefore, **B** is the Smith set candidate with the best AV score.

Benham: There is no Condorcet winner, so we eliminate A, who is the plurality loser. B is a Condorcet winner among the remaining candidates, so **B** wins.

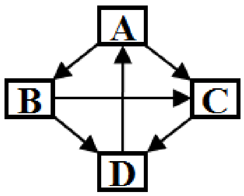
Smith-AV: D is not in the Smith set, so he is eliminated. C is eliminated in the first AV counting round, and B is eliminated in the second AV counting round, so **A** is the winner.

Tideman: This rule works the same as Smith-AV in this example, and thus elects **A**. In the last phase, B is eliminated because he is no longer in the Smith set rather than because he is the plurality loser, but with only two candidates remaining, these are equivalent.

Example 2: Benham and Tideman differ from Woodall and Smith-AV

4	ABCD
5	BDAC
6	CDAB

P	A	B	C	D
A		10	9	4
B	5		9	9
C	6	6		10
D	11	6	5	



	r1		r2		r3	
A	4		4	X	-	
B	5		5	+4	9	✓
C	6		6		6	X
D	0	X	-		-	

AV or Smith-AV tally

	r1		r2	
A	4		4	✓
B	5		5	X
C	6		6	X
D	0	X	-	

Benham or Tideman tally

Woodall: In an AV tally, D is eliminated first, followed by A, and then C, leaving B as the winner. Therefore, **B** is the Smith set candidate with the best AV score.

Benham: There is no Condorcet winner, so we eliminate D who is the plurality loser. A is the Condorcet winner among remaining candidates, so **A** wins.

Smith-AV: All candidates are in the Smith set, so we proceed to the AV tally. D has no first-choice votes, so he is eliminated in the first AV counting round. In the second AV round, A has 4 first choice votes, B has 5, and C has 6, so A is eliminated. In the third AV round, C is eliminated, and **B** wins.

Tideman: All candidates are in the Smith set. The plurality loser is D, so he is eliminated. Recalculating the Smith set, we find that A is now the Condorcet winner, so **A** wins.

5. Strategic Voting

There is no single, agreed way to measure vulnerability to strategic voting, but one approach is to simulate elections using a specified data-generating process, and then to determine the percentage of trials in which coalitional manipulation is possible in each method.[17] That is, in what percentage of trials does there exist a group of voters who all prefer another candidate to the sincere winner, and who can cause that candidate to win by changing their votes?

Here, I will present results arising from two data generating processes: a spatial model, and an impartial culture model. I recognize that this is not exhaustive, as there are an infinite number of possible data generating processes, but it will serve at least to give preliminary evidence, and to demonstrate some basic principles.[18]

The spatial voting model used here distributes both voters and candidates randomly in N-dimensional issue space, according to a multivariate normal distribution without covariance. Voters are then assumed to prefer candidates who are closer to them in this issue space. Formally,

$$L_{vn} \sim \mathcal{N}(0,1), \forall v, n.$$

$$\Lambda_{cn} \sim \mathcal{N}(0,1), \forall c, n.$$

$$U_{vc} = -\sqrt{\textstyle\sum_{n=1}^{N} (L_{vn} - \Lambda_{cn})^2}, \forall v, c.$$

(The L and Λ matrices give the voter and candidate locations, respectively.)

The impartial culture model used here simply treats each voter's utility over each candidate as an independent draw from a uniform distribution, thus making each ranking equally probable, independent of other voters' rankings. Formally, $U_{vc} \sim \mathcal{U}(0,1), \forall v, c$.

In order to avoid massive computational cost, I make the restrictive assumption that all voters in the strategic coalition must cast the same ballot. Thus, I am not computing the

[17] For example, see Chamberlin (1985), Lepelley and Mbih (1994), Kim and Roush (1996), Favardin, Lepelley, and Serais (2002), Favardin and Lepelley (2006), Tideman (2006), and Green-Armytage (2011).

[18] Green-Armytage (2011) also uses the voter ratings of politicians in the American National Election Studies time series survey as a data generating process, and finds that it gives similar results to the models used here.

frequency with which manipulation is possible, but rather finding a lower-bound approximation.[19]

Tables 1 and 2 show the results of this analysis, given various specifications of the spatial model and the impartial culture model, respectively. I use 10,000 trials for each specification, which causes the margin of error to be .0098 or less,[20] with 95% confidence. In addition to applying the analysis to Woodall, Benham, Smith-AV, and Tideman, I apply it to AV, ranked pairs, beatpath, plurality,[21] minimax,[22] Borda,[23] approval voting,[24] and range voting.[25] Figures 1 and 2 illustrate a subset of these results. To make the graphs less convoluted, I allow Woodall to stand in for the other three Condorcet-Hare hybrids, I allow minimax to stand in for ranked pairs and beatpath, and I allow approval voting to stand in for range voting.

In every one of these specifications, the five methods that are least frequently manipulable are Woodall, Benham, Smith-AV, Tideman, and AV. Among these methods, AV is vulnerable with slightly greater frequency, but the difference tends to be very small. Likewise, there are some specifications in which Woodall and Benham outperform Smith-AV and Tideman, but their scores are usually extremely close or identical. Minimax, beatpath, and ranked pairs are all vulnerable with substantially greater frequency than these five, but they are all vulnerable with substantially lower frequency than plurality, which in turn is vulnerable with substantially lower frequency than Borda, approval, and range.

One notable feature of the spatial model is that vulnerability is substantially higher across the board when $N = 1$, and that it decreases rapidly as N increases. Given $N > 1$, the difference between the best five methods and the remaining methods is particularly striking. One notable feature of the impartial culture model is that although the probability that a method will be vulnerable to manipulation seems to converge to 100% as V becomes large for all of the other methods included here, it doesn't do so for AV and the Smith-AV hybrids.

Why are AV and the Condorcet-Hare hybrids vulnerable with lower frequency than the other methods? To give some intuition for this, it may be helpful to define two particular types of strategic voting: 'compromising' and 'burying'. Suppose that w is the sincere winner, and q is an alternative candidate whom strategic voters are seeking to elect instead. In this context, the **compromising strategy** would be their giving q a better ranking (or rating), and the **burying strategy** would be their giving w a worse ranking (or rating).[26] Together, these tactics seem to account for most strategic possibilities.[27]

[19] Green-Armytage (2011) performs calculations that don't rely on this assumption, but these calculations are not applied to any Condorcet-Hare hybrid meth+ods. Doing so without massive computational cost presents a set of interesting programming challenges. Meanwhile, comparing the results from the two papers suggests that the assumption of uniform strategic coalitions has only a minor impact on the manipulability of most methods.

[20] A margin of error of ±.0098 is the upper bound, which applies when the true probability is exactly one half. I further reduce the random error in the *difference* between the scores that the various voting methods receive by using the same set of randomly generated elections for each method.

[21] I define the plurality winner as the candidate with the most first choice votes.

[22] The minimax winner is the Condorcet winner if one exists, or otherwise, the candidate whose worst loss is least bad. Formally:

$$\mathrm{M}_y = \max_{x=1}^{C} P_{xy} - \tau_y, \forall y = 1, \ldots, C.$$

$$w = \mathrm{argmin}(\mathrm{M}).$$

[23] The Borda winner is the candidate with the most points, if each first choice vote is worth C points, each second choice vote is worth $C - 1$ points, and so on. Equivalently, Borda can be calculated as follows:

$$\mathrm{B}_y = \sum_{x=1}^{C} P_{xy} - \tau_y, \forall y = 1, \ldots, C.$$

$$w = \mathrm{argmin}(\mathrm{B}).$$

[24] Each voter can give each candidate either one point or zero points. The winner is the candidate with the most points.

[25] Each voter can give each candidate any number of points in a specified range, e.g. 0 to 100. The winner is the candidate with the most points.

[26] The terms 'compromising' and 'burying' were used by Blake Cretney in the currently-defunct web site condorcet.org.

[27] This is somewhat intuitive, and supporting evidence is given in Green-Armytage (2011).

Table 1: Strategic voting, spatial model

V	*N*	*C*	Woodall	Benham	Smith-AV	Tideman	AV	minimax	beatpath	ranked pairs	plurality	Borda	approval	range
99	1	3	.140	.140	.140	.140	.140	.152	.152	.152	.282	.395	.549	.594
99	1	4	.325	.325	.335	.335	.325	.351	.359	.359	.549	.825	.798	.865
99	1	5	.487	.487	.512	.512	.487	.550	.560	.560	.732	.980	.912	.960
99	1	6	.622	.622	.660	.660	.622	.694	.707	.707	.844	.998	.960	.988
99	2	3	.038	.038	.038	.038	.045	.191	.189	.189	.229	.424	.500	.500
99	2	4	.104	.104	.107	.107	.119	.358	.359	.359	.492	.734	.731	.785
99	2	5	.186	.186	.194	.194	.209	.490	.492	.492	.693	.900	.840	.905
99	2	6	.262	.262	.279	.279	.287	.599	.601	.601	.825	.964	.903	.961
99	3	3	.019	.019	.019	.019	.026	.192	.192	.192	.212	.426	.470	.468
99	3	4	.044	.044	.044	.044	.059	.333	.333	.333	.440	.707	.684	.733
99	3	5	.077	.077	.080	.080	.100	.431	.431	.431	.617	.854	.796	.861
99	3	6	.116	.116	.122	.123	.146	.520	.521	.521	.765	.927	.871	.933
99	4	3	.013	.013	.013	.013	.020	.198	.198	.198	.210	.426	.463	.457
99	4	4	.031	.031	.032	.032	.044	.321	.321	.321	.419	.697	.668	.710
99	4	5	.048	.048	.049	.049	.068	.413	.413	.413	.599	.835	.779	.848
99	4	6	.065	.065	.068	.068	.091	.478	.480	.480	.726	.908	.854	.915
99	16	3	.002	.002	.002	.002	.006	.186	.183	.183	.187	.416	.432	.431
99	16	4	.007	.007	.007	.007	.015	.290	.291	.291	.369	.653	.629	.658
99	16	5	.010	.010	.010	.010	.020	.350	.350	.350	.497	.770	.733	.772
99	16	6	.014	.014	.014	.014	.028	.399	.398	.398	.601	.843	.807	.845

Table 2: Strategic voting, impartial culture model

V	*C*	Woodall	Benham	Smith-AV	Tideman	AV	minimax	beatpath	ranked pairs	plurality	Borda	approval	range
9	3	.101	.101	.101	.101	.119	.357	.342	.342	.389	.560	.599	.606
9	4	.213	.213	.216	.216	.235	.557	.557	.557	.625	.794	.775	.829
9	5	.307	.307	.313	.314	.333	.682	.694	.694	.763	.897	.858	.910
9	6	.389	.389	.402	.403	.419	.763	.781	.781	.847	.943	.911	.952
29	3	.099	.099	.099	.099	.126	.681	.676	.676	.694	.816	.837	.843
29	4	.188	.188	.188	.188	.231	.846	.845	.845	.921	.965	.952	.976
29	5	.282	.282	.285	.285	.335	.912	.914	.914	.981	.989	.981	.996
29	6	.355	.355	.362	.362	.415	.948	.948	.948	.995	.995	.993	.998
99	3	.088	.088	.088	.088	.123	.951	.952	.952	.951	.989	.986	.990
99	4	.180	.180	.180	.180	.241	.987	.987	.987	.999	.998	.999	1.000
99	5	.255	.255	.255	.255	.327	.995	.995	.995	1.000	.993	1.000	1.000
99	6	.312	.312	.312	.312	.405	.998	.998	.998	1.000	.979	1.000	1.000

Figure 1: Strategic voting, spatial model, N=4, V=99

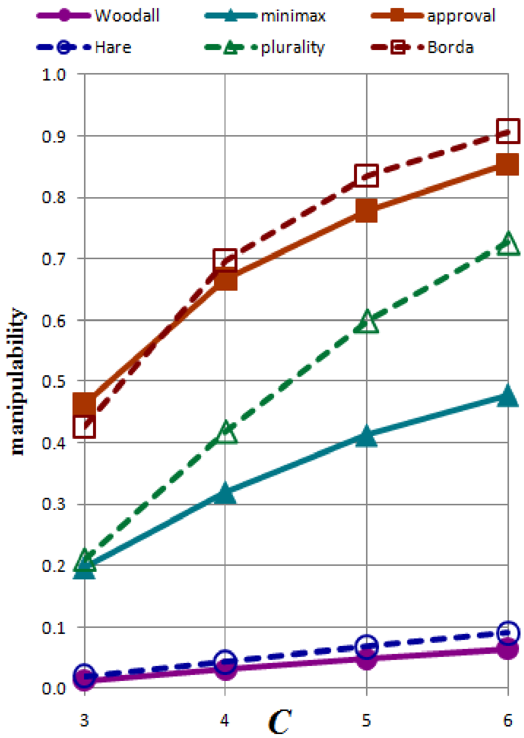
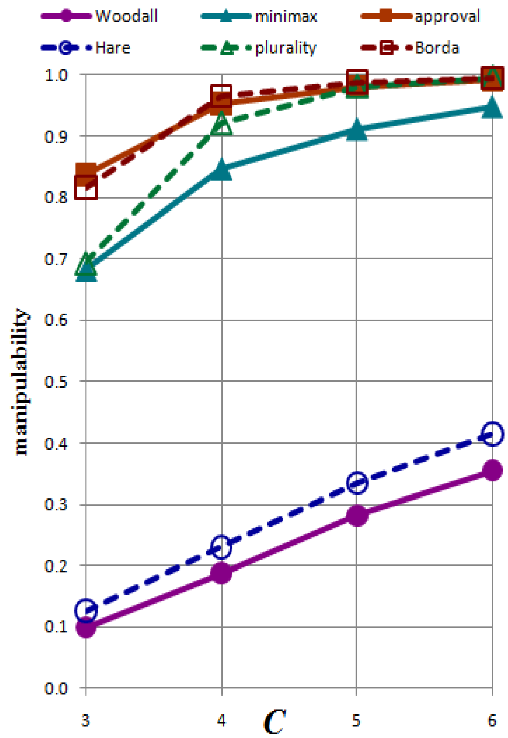


Figure 2: Strategic voting, impartial culture model, V=29



AV is immune to the burying strategy, and it is only vulnerable to the compromising strategy in relatively rare situations, such as when the AV winner and Condorcet winner are different, or when there is a majority rule cycle. The Condorcet-Hare hybrids are strictly less vulnerable to compromising, in that they are only vulnerable when there is a majority rule cycle. All Condorcet-efficient methods are vulnerable to burying,[28] but this vulnerability seems to be substantially less frequent in the Condorcet-Hare hybrids than in most other Condorcet methods. The reason for this is that voters who prefer q to w will already have ranked q ahead of w, so that further burying w will not affect w's plurality score unless q has already been eliminated. Burying w can create a cycle with q and some other candidate or candidates, but unless w already happens to be the plurality loser among the candidates in this cycle, the strategy is unlikely to actually elect q.

[28] Woodall (1997) demonstrates that Condorcet is incompatible with the properties of 'later-no-help' and 'later-no-harm', which is a nearly equivalent statement.

6 Strategic Nomination

A comparable method can be applied to measuring the frequency of incentives for strategic nomination, which I define here as non-winning candidates entering or leaving the race in order to change the results to ones they prefer.[29] For example, suppose that A wins given the set of candidates {A,B,C}, but B wins given the set {A,B}, and candidate C prefers B to A. In this case, candidate C has an incentive for **strategic exit**. Alternatively, suppose that X wins given the set of candidates {X,Y}, but Y wins given the set {X,Y,Z}, and Z prefers Y to X. In this case, candidate Z has an incentive for **strategic entry**.

I use only the spatial model for my strategic nomination analysis here, because it provides the more straightforward method of determining candidates' preferences over other candidates; that is, it is natural to imagine that candidates prefer other candidates who are closer to them in the issue space. Formally,

[29] This analysis follows Green-Armytage (2011).

$\Psi_{xy} = -\sqrt{\sum_{n=1}^{N}(\Lambda_{xn} - \Lambda_{yn})^2}$ gives the utility of candidate x if candidate y wins (and vice versa).

Aside from V and N, the parameters of the model are CI and CO, which represent the number of candidates who are initially in the race (but who have the ability to exit), and the number of candidates who are initially out of the race (but who have the ability to enter).

I exclude approval and range from this analysis, because any effects that show up will only be an artefact of the way that utilities are transformed into approval votes and range scores, respectively. If this transformation is independent of which candidates are actually running, then nomination vulnerability is always zero.

Tables 3 and 4, and figures 3 and 4, present the result of some strategic nomination simulations, once again with 10,000 trials per specification.

The most salient result here is that all of the Condorcet methods are only slightly vulnerable to both strategic exit and strategic entry, while other methods are more vulnerable. Plurality is highly vulnerable to strategic exit; presumably, this helps to explain the common practice of holding party primaries so that candidates with similar ideologies don't get in each others' way. AV is substantially vulnerable to strategic exit as well, especially when C is large. Borda is the most vulnerable to strategic entry.

Condorcet methods are vulnerable to strategic exit only if there is a majority rule cycle among the candidates who are in the race; if there is a Condorcet winner to begin with, he will remain the Condorcet winner after the deletion of any other candidate.[30] Likewise, they are vulnerable to strategic entry only if there is a cycle when the newly-entered candidate is included. In the spatial model, majority rule cycles are rare, so Condorcet methods are rarely vulnerable to strategic nomination.

[30] Note that the existence of a cycle doesn't necessarily imply an incentive for strategic exit, though it does imply an incentive for strategic voting.

7 Mathematical Properties

We will see in this section that the four Condorcet-Hare hybrids are similar enough to have the same status with respect to most mathematical properties. Like all other Condorcet-efficient rules, they lack participation,[31] and like AV, they lack monotonicity as well. Meanwhile, they possess Smith consistency, along with properties that are implied by this, such as Condorcet, Condorcet loser,[32] strict majority,[33] and mutual majority.[34]

While thus sharing many properties, these methods can nevertheless be distinguished on the basis of lesser-known (and arguably less significant) properties. For example, Smith-AV and Tideman have a property called 'Smith-IIA', but lack two properties called 'mono-add-plump' and 'mono-append', whereas for Woodall and Benham, the opposite is true.

7.1. Monotonicity

Definition: If x is not the winner, then changing ballots by giving x an inferior ranking will never change the winner to x. (Conversely, if x is the winner, then changing ballots by giving x a superior ranking will never cause x to lose.)

Example 3: Woodall, Benham, Smith-AV, Tideman, and AV all lack monotonicity

7	ABC
10	BCA
6	CAB

Given any of the five systems, the initial winner is A, but if two of the BCA voters change their votes to CBA, the winner will change to B.

[31] Moulin (1988) demonstrates that no method can simultaneously possess Condorcet consistency and the participation property.

[32] A Condorcet loser is a candidate who loses all pairwise comparisons. The Condorcet loser property states that such a candidate never wins.

[33] This property states that if candidate x is ranked first by a majority of voters, then x is elected.

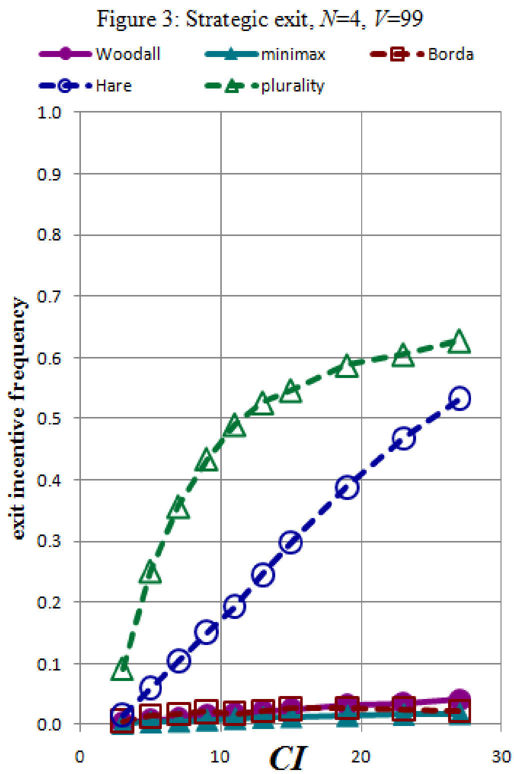
[34] This property states that if there is a set of candidates such that a cohesive majority of voters ranks all members in the set ahead of all members outside the set, then the winner is a member of the set.

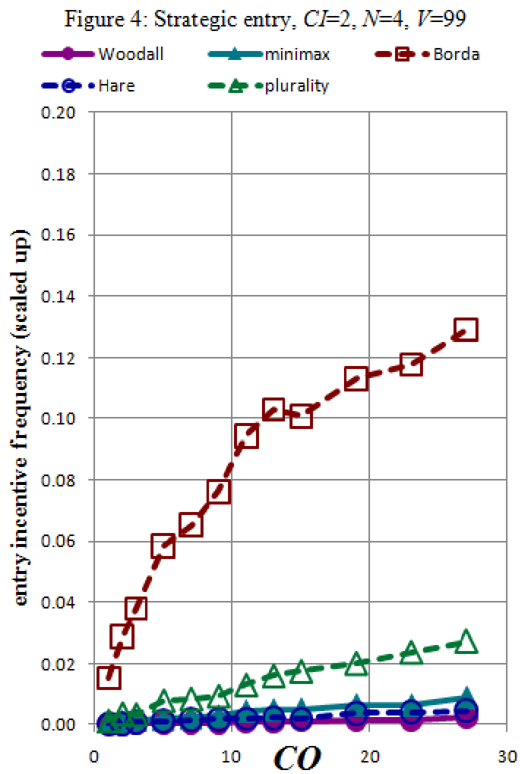
Table 3: Strategic exit

V	*S*	*CO*	*CI*	Woodall	Benham	Smith-AV	Tideman	AV	minimax	beatpath	ranked pairs	plurality	Borda
99	4	0	3	.002	.002	.002	.002	.015	.001	.001	.001	.091	.006
99	4	0	5	.007	.007	.007	.007	.060	.004	.003	.004	.251	.013
99	4	0	7	.010	.010	.010	.010	.104	.006	.005	.005	.356	.017
99	4	0	9	.015	.015	.015	.015	.151	.008	.008	.009	.434	.021
99	4	0	11	.018	.017	.018	.018	.193	.010	.009	.010	.490	.018
99	4	0	13	.022	.021	.022	.021	.245	.012	.012	.012	.526	.022
99	4	0	15	.025	.025	.025	.025	.298	.013	.013	.014	.546	.026
99	4	0	19	.033	.030	.032	.030	.389	.015	.013	.014	.588	.027
99	4	0	23	.036	.033	.035	.034	.468	.017	.016	.016	.605	.026
99	4	0	27	.041	.039	.040	.038	.533	.018	.018	.018	.627	.022

Table 4: Strategic entry

V	*S*	*CO*	*CI*	Woodall	Benham	Smith-AV	Tideman	AV	minimax	beatpath	ranked pairs	plurality	Borda
99	4	1	2	.000	.000	.000	.000	.000	.001	.001	.001	.001	.015
99	4	2	2	.000	.000	.000	.000	.000	.001	.001	.001	.004	.029
99	4	3	2	.000	.000	.000	.000	.001	.002	.002	.002	.003	.038
99	4	5	2	.000	.000	.000	.000	.001	.002	.002	.002	.008	.059
99	4	7	2	.001	.001	.001	.001	.002	.002	.003	.003	.009	.065
99	4	9	2	.000	.000	.000	.000	.002	.003	.003	.003	.009	.076
99	4	11	2	.001	.001	.001	.001	.002	.005	.005	.005	.013	.094
99	4	13	2	.001	.001	.001	.001	.002	.005	.005	.005	.016	.103
99	4	15	2	.001	.001	.001	.001	.002	.005	.005	.005	.018	.101
99	4	19	2	.002	.002	.002	.002	.004	.006	.007	.007	.020	.113
99	4	23	2	.001	.001	.001	.001	.004	.006	.006	.006	.024	.118
99	4	27	2	.003	.003	.003	.003	.005	.009	.008	.008	.027	.129





7.2. Participation

Definition: If the initial winner is x, and an extra vote is added that ranks x ahead of y, it will never change the winner to y.
Discussion: To lack this property is also known as the no-show paradox. This property is closely related to another property, known variously as consistency,[35] separability,[36] and combinativity,[37] which states that if x is the winner according to each of two separate sets of ballots, then x will be the winner when the sets are combined.

Example 4: Woodall, Benham, Smith-AV, Tideman, and AV all lack participation

4	ABC
5	BCA
6	CAB

Assume that ties are broken lexicographically. Given any of the four systems, the initial winner is B, but adding another ABC voter changes the winner to C.

7.3. Mono-add-plump[38]

Definition: If x is the winner, and one or more ballots are added that rank x first, and indicate no further rankings, then x will necessarily remain the winner.

Discussion: This property can be thought of as a weaker version of the participation property or the consistency property.

Example 5: Smith-AV and Tideman lack mono-add-plump

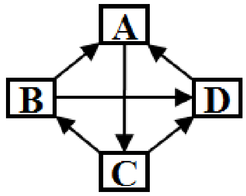
8	ACBD
3	BACD
7	CBDA
5	DBAC

[35] In Young (1975).
[36] In Smith (1973).
[37] In Tideman (2006).

[38] This property is defined in Woodall (1996), along with mono-append below. I credit Chris Benham with pointing out that these properties provide a distinction between Woodall and Benham on one hand, and Smith-AV and Tideman on the other.

Tallies for Example 5 without and with added ballots

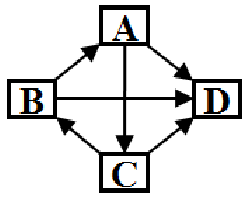
P	A	B	C	D
A		8	16	11
B	15		8	18
C	7	15		18
D	12	5	5	



	round 1		round 2		round 3	
A	8	+3	11	+5	16	✔
B	3	X	-		-	
C	7		7		7	X
D	5		5	X	-	

Smith-AV or Tideman tally

P	A	B	C	D
A		10	18	13
B	15		8	18
C	7	15		18
D	12	5	5	



	round 1		round 2		round 3	
A	10		10		10	X
B	3	+5	8	+7	17	✔
C	7		7	X	-	
D	5	X	-		-	

Smith-AV or Tideman tally

Given these ballots, A will win under both Smith-AV and Tideman. However, adding two voters who only indicate a first preference for A will change the winner to B. (Adding the A-only votes removes D from the Smith set, which in turn strengthens B.) The tallies are presented above, first without the extra ballots, and then with them.

Proposition 1: Woodall possesses mono-add-plump

Proof:
1. Suppose that with the original set of ballots, candidate *x* wins in round *r*. That is, if the Smith set has any members other than *x*, they are eliminated before round *r* in the AV count.
2. Adding *x*-only ballots will not affect the order in which candidates are eliminated in any round before *r*.
3. Adding *x*-only ballots will not remove *x* from the Smith set.
4. Adding *x*-only ballots will not add new candidates to the Smith set.
5. In view of 2–4, adding *x*-only ballots can't prevent candidate *x* from winning in round *r*. ■

Proposition 2: Benham possesses mono-add-plump

Proof:
1. Suppose that with the original set of ballots, candidate *x* wins in round *r*. That is, as of round *r*, *x* is a Condorcet winner among the remaining candidates.
2. Adding *x*-only ballots will not affect the order in which candidates are eliminated in any round before *r*. Therefore, the set of non-eliminated candidates in round *r* will not be changed.
3. If *x* is a Condorcet winner among a given set of candidates, adding *x*-only ballots will not change this.
4. In view of 2 and 3, adding *x*-only ballots can't prevent candidate *x* from winning in round *r*. ■

Table 5: overall summary

	Woodall	Benham	Smith-AV	Tideman	AV	beatpath, ranked pairs	minimax	plurality	Borda	approval, range

Smith	✔	✔	✔	✔	X	✔	X	X	X	X
HRSV	✔	✔	✔	✔	✔	X	X	X	X	X
HRSN	✔	✔	✔	✔	X	✔	✔	X	X	✔
monotonicity	X	X	X	X	X	✔	✔	✔	✔	✔
participation	X	X	X	X	X	X	X	✔	✔	✔
Condorcet	✔	✔	✔	✔	X	✔	✔	X	X	X
Condorcet loser	✔	✔	✔	✔	✔	✔	✔	X	✔	X
strict majority	✔	✔	✔	✔	✔	✔	✔	✔	X	X
mutual majority	✔	✔	✔	✔	✔	✔	X	X	X	X
Smith-IIA	X	X	✔	✔	X	✔	X	X	X	X
MAP/MA	✔	✔	X	X	✔	✔	✔	✔	✔	✔

7.4. Mono-append

Definition: If *x* is the winner, and one or more ballots that previously left *x* unranked are changed only in that *x* is added to the ballot after the last ranked candidate, then *x* will necessarily remain the winner.

Discussion: This property is fairly similar to mono-add-plump.

Example 6: Smith-AV and Tideman lack mono-append

10 ACBD
3 B
7 CBDA
5 DBAC

With these ballots, A will win both Smith-AV and Tideman. However, changing the 3 B votes to BA votes will change the winner to B. (Again, this strengthens B by removing D from the Smith set.)

It is fairly easy to see that Woodall and Benham possess mono-append, following logic similar to that of the proofs of propositions 1 and 2 above.

7.5. Smith-IIA[39]

Definition: Removing a candidate from the ballot who is not a member of the Smith set will not change the result of the election. (The 'IIA' here stands for 'independence of irrelevant alternatives'.)

Example 1 above shows that Woodall and Benham lack this property. That is, removing D will change the winner from B to A.

It is easy to see that Smith-AV and Tideman both possess this property, because both methods begin by eliminating candidates outside the Smith set.

8 Conclusion

Table 5 summarizes the results from sections 5–7. HRSV and HRSN are abbreviations for 'highly resistant to strategic voting', and 'highly resistant to strategic nomination'. (Of course, reducing the simulation results to a binary score requires the imposition of a somewhat arbitrary cut-off, but in general, the methods deemed 'highly resistant' in each category perform substantially better than the others.) MAP/MA is an abbreviation for mono-add-plump and mono-append.

Woodall, Benham, Smith-AV, and Tideman possess Smith consistency (and therefore the Condorcet, Condorcet loser, strict majority, and mutual majority properties), and offer relatively few opportunities for strategic voting and strategic nomination; I suggest that this combination of properties could be valuable if applied to single-winner public elections. I don't conclude that any of these methods is unambiguously better than the others; rather, I

[39] Defined in Schulze (2003).

leave it to the reader to decide which one he or she prefers.

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About the Author

James Green-Armytage is Visiting Instructor in the Economics Department at Virginia Tech. He was born in New York City, where he attended the Hunter College Campus Schools. He earned a B.A. from Antioch College in 2004.

Party Lists and Preference Voting

I.D. Hill
d.hill928@btinternet.com

Abstract

Elections by party lists, where voting is just by choosing a single party, can lead to unrepresentative results because of wasted votes. A system is suggested that would allow voting by preference rankings for parties. It is suggested that this would be an improvement.

Keywords: elections; party lists; European Parliament; wasted votes; preference voting; STV

1 Introduction

In 2009, following the election of two candidates representing the British National Party (BNP) to the European Parliament, the BBC felt bound to treat that party with impartiality and invited its leader to be on the panel for an edition of its “Question Time” television program. There followed much public criticism of the party. There was also criticism, unfair in my opinion, of the BBC for doing so. Yet there seems to have been no such criticism of the “closed party list” rules by which European Parliament elections are currently conducted in the United Kingdom (except Northern Ireland who use STV), or of Jack Straw, who was also a member of that panel, and who, as the relevant member of the Government at the time, had been responsible for forcing those rules through Parliament. They certainly played a part in allowing the BNP to take those seats.

I strongly support the right of the electorate to elect whom they wish, whether or not I personally approve of those individuals or of their parties, but that does assume that the electoral system used was one that reasonably represented that electorate. I do not believe that a party list system is capable of doing so but, so long as it is difficult to move politicians away from party lists where they are already in force, it is worth considering how party-list voting systems might be improved.

Party lists force voters to consider political party as of major importance whether they wish to do so or not, thus increasing the already excessive power of party organisations. Apart from that, the main trouble is that party-list voting is just by an X for a single party, which has the same disadvantage of wasted votes that an X vote has when voting for individual candidates.

2 European Parliament Election 2009[1]

There were two constituencies where a BNP candidate was elected. In the “North West” constituency there were 12 parties standing, plus 1 independent candidate, who counts for these purposes as a 13th party. In the “Yorkshire and the Humber” constituency there were the same 12 parties but no independent candidate. In order of their numbers of votes, they were as shown in Table 1. In the remainder of this paper I shall concentrate on just the North West constituency. The arguments would be just the same for either.

The results were determined using the d'Hondt system. In the North West constituency, there were elected 3 Conservatives, 2 Labour, 1 Liberal Democrat, 1 UKIP and 1 BNP. The BNP candidate got 132094 votes out of a total of 1651825. Now if 132094 are enough to secure a seat, all 8 seats need a total of 8 times that or 1056752,

[1] See Mellows-Facer et al. (2009) [1] for details of the election.

Table 1: Results of the European Parliament Election of 2009 in Two Constituencies

		North West	Yorkshire and Humber
Conservative Party	[Cons]	423174	299802
The Labour Party	[Lab]	336831	230009
United Kingdom Independence Party	[UKIP]	261740	213750
Liberal Democrats	[LibD]	235639	161552
British National Party	[BNP]	132094	120139
The Green Party	[Gree]	127133	104456
English Democrats Party	[Engl]	40027	31287
Socialist Labour Party	[SLP]	26224	19380
Christian Party “Proclaiming Christ's Lordship”	[Chri]	25999	16742
No2EU: Yes to Democracy	[NoEU]	23580	15614
Jury Team	[Jury]	8783	7181
Pro Democracy: Libertas	[ProD]	6980	6268
Independent: Francis Apaloo	[Ind]	3621	

indicating wasted votes of 1651825 – 1056752 = 595073. That is to say wasted votes were more than 4 times what the BNP got, or 36% of the total, compared with the unavoidable wastage of 1 Droop quota which, for 8 seats, is just over 11% This wastage consists of the votes for parties that did not achieve a seat, plus the surplus votes of those that did achieve one or more seats.

Could anything be done to reduce this wastage? What I should wish to see would be the complete replacement of the party list method with STV for individual candidates, who could bear party labels if they wished of course. In fairness we should note that under STV, assuming the same number of votes, the quota would be 183537, so the BNP attained 72% of a quota. If those were translated into first preferences it is quite possible that transfers would have enabled BNP to take a seat, but if they were capable of taking a seat on a fair electoral system, then so be it. The electorate have the right to choose what they want.

3 Preference voting

But if we assume that no Westminster Government is likely to enact STV in the near future, can anything be done that would partially rectify the situation? Even restricting voters to voting only for parties, not individuals, it would be an improvement to let each voter list the parties in order of preference, so as to allow redistribution of the votes, as in STV.

In investigating how this might work in practice, we face the difficulty that we do not know what the voters' preferences would be. To some extent we can guess, where the big well-known parties are concerned, but even that is difficult for the minor parties where we know little about them and what they stood for. Nor do we know whether voters for a big party would prefer even a rival big party to a small party, and sometimes parties that have similar aims, or similar names, are nevertheless bitterly opposed to one another. The work of Clarke et al. (2010) [2] can help to some extent, but it tells us nothing about the minor parties. Nor can we assume that what voters say that they would do for a second preference among the bigger parties represents what they would do for a seventh preference, say. We can but guess, taking [2] into account where possible, but I need to make clear that what I have assumed does not represent what the real voters would actually have done. I have also had to guess what proportion of voters would express further preferences at each stage, and what proportion would stop short of a full listing.

The proposed voting procedure is similar in principle to STV, except that when a party attains a seat, it is still possible for it to attain further seats, so it keeps its surplus in hope of doing so. I have calculated the result using Meek's method, but other versions of STV could be used if desired.

Introducing some new terminology, I define a party's “balance” to mean its total votes if it has not yet attained a seat, or to mean its surplus if it has attained one or more seats. I define a party being “dormant” to mean excluded if not having attained a seat, or to mean not allowed to take any further seat if having already attained any. Using Meek rules implies that a party can accept further votes, to be passed on to other parties in fair proportion, even after becoming dormant.

The plan, then, is to treat a vote for ABC, say, where A, B and C are parties that each have a list of 4 candidates, as if it had been a vote for the individual candidates A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4, in that order, and use STV with one modification: that when an exclusion is necessary, all of the non-elected candidates of the party that has the smallest balance are excluded together, and the party becomes dormant.

4 An example

Table 2 shows how this could work, taking the actual votes for the various parties and my assumptions as to how the transfers might go. I must emphasise that this is for illustrative purposes only. It is not intended to show what would actually have happened, which we can never know.

While each party, except the Independent, had a list of eight candidates, the table shows only those who will be actually involved in the count The figures are shown rounded to integers for simplicity, though the calculations were actually to more figures.

At stage 1 we have first preferences to match the actual result. Four candidates have passed the quota and are marked E (for Elected). At stage 2 their surpluses have been moved on to the next in the list; the second Conservative candidate has now also passed the quota and is elected.

At stage 3 nobody can be elected so the Independent candidate is excluded and his votes are redistributed in stage 4. Similarly at stages 4 - 9, but it is seen that the Christian Party has overtaken the Socialist Labour Party, so SLP goes out at stage 7, although having had more first preferences.

Up to stage 9 the Xs indicate the exclusion of a whole party, but thereafter the Xs indicate the exclusion of the candidates shown, their parties becoming dormant, while keeping their elected candidates.

In the later stages a candidate is sometimes elected before the iteration to the final result of the stage is complete, so the already-elected candidates have more than a quota. This is not incorrect.

The hypothetical result shows the Conservatives and BNP as each having lost a seat compared with what actually happened, with UKIP and the Green Party taking them instead, but it must be stated again that this is wholly hypothetical. It would be perfectly easy to make up supposed transfers that let the BNP take a seat after all.

Given the huge wastage of votes in what actually happened, I suggest that the proposed system would have been likely to achieve results that better represented the wishes of the voters. I say again, though, that the aim of a system should be to represent what the voters want, not to support or oppose any particular party.

5. Conclusion

It may be noted that, had the Sainte-Laguë system been used instead of the d'Hondt system, a Green candidate would have been elected instead of the third Conservative candidate, but I regard argument about the merits of Sainte-Laguë compared with the merits of d'Hondt, while ignoring the question of wasted votes, as noticing the mouse but not the elephant.

The referee has suggested that a simple way to avoid the perceived problem of electing extreme parties is simply to have smaller constituencies. That would, indeed, make the election of extreme parties less likely, but it would be likely to increase rather than diminish the number of wasted votes. To avoid misunderstanding, I must emphasise that the aim of this paper is not to show that extreme parties would not be elected with preferential voting. It is merely to examine whether a system is possible, within a party list

Table 2. Hypothetical Results for the North West Constituency under the Proposed System**.**

Stage	1	2	3	4	5	6	7	8
Cons1	423174 E	183536	183536	183425	183214	182949	182539	182049
Cons2		239638 E	183536	183425	183214	182949	182539	182049
Cons2		239638 E	183536	183425	183214	182949	182539	182049
Cons3			56102	56526	57246	58576	60294	61691
Lab1	336831 E	183536	183536	183425	183214	182949	182539	182049
Lab2		153295	153295	153607	154117	155282	156492	165800
Lab3								
UKIP1	261740 E	183536	183536	183425	183214	182949	182539	182049
UKIP2		78204	78204	78516	79026	79790	89999	90904
LibD1	235639 E	183536	183536	183425	183214	182949	182539	182049
LibD2		52103	52103	52415	52925	54090	55300	62637
BNP1	132094	132094	132094	132094	132194	132294	132394	132809
Gree1	127133	127133	127133	128040	130036	132834	136733	141500
Gree2								
Engl1	40027	40027	40027	40128	40526	41025	44624	45039
SLP1	26224	26224	26224	26325	26524	26824	26924 X	
Chri1	25999	25999	25999	26100	26399	26898	27398	27813 X
NoEU1	23580	23580	23580	23681	23880	24180 X		
Jury1	8783	8783	8783	9185	9983 X			
ProD1	6980	6980	6980	7081 X				
Ind1	3621	3621	3621 X					
n/t	0	0	0	1003	2899	5288	8971	13388
quota	183536	183536	183536	183425	183214	182949	182539	182049

Stage	8	9	10	11	12	13	14	15
Cons1	182049	181488	180631	179433	179391	179366	190075	177500
Cons2	182049	181488	180631	180290	179391	179366	190075	179962
Cons3	61691	64977	70261 X					
Lab1	182049	181488	180631	179483	179391	179366	194115	177387
Lab2	165800	169969	174501	180278 E	179391	179366	194115	179360
Lab3					1039 X			
UKIP1	182049	181488	180631	179827	179391	179366	186775	178871
UKIP2	90904	93833	104461	137756	138619	138829	144564	177046 E
LibD1	182049	181488	180631	179670	179391	179366	179365	178302
LibD2	62637	70104	80087	98171	98676	99276 X		
BNP1	132809	133015	134800	137332	137365	137378	138639	140201
Gree1	141500	148188	158414	162422	162474	162615	181044 E	178613
Gree2								14582
Engl1	45039	45864 X						
Chri1	27813 X							
n/t	13388	18435	26146	37163	37307	37532	53055	69998
quota	182049	181488	180631	179407	179391	179366	177641	175759

context, that would better represent what the voters actually want, whether that includes extremist parties or not.

I continue to dislike in principle anything of a party list nature, following Enid Lakeman's dictum that party should matter only to the extent that voters wish it to matter. Still, a small improvement in representativeness achieved by reducing wasted votes is better than no improvement at all.

6 Acknowledgements

Discussion with Simon Gazeley about an earlier version of this proposal greatly improved it. I also thank the referee whose remarks on my first submission were very useful, and who brought reference [2] to my attention.

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About the Author

David Hill is a retired statistician. He was formerly a member of Council of the Electoral Reform Society and chairman of its Technical Committee.

The Matrix Vote: Electing an all-party coalition cabinet

Peter Emerson
pemerson@deborda.org

Abstract

One method of electing the cabinet of a coalition government is the matrix vote, the outcome of which is (almost) bound to be proportional to party support, with, potentially, each minister serving in that position for which those voting think he/she is most suited. This article discusses the concept of the matrix vote, describes an experiment that was conducted to see how it might work, and assesses its practical implications.

Keywords: Borda, consensus, matrix vote, power-sharing

1 Introduction

The matrix vote is a form of proportional representation that uses voters' ranked preferences not only to determine a set of winning candidates but also to assign them to specified positions. Unlike other forms of proportional representation, therefore, the matrix vote ballot requires that voters report their choices in two dimensions. In the first dimension, every voter may rank as many candidates as there are positions; in the second dimension the voter specifies his/her choice of a position for each ranked candidate. The votes are then used in two election counts: the first to determine who has been elected, the second to assign each successful candidate to a position. The matrix vote could be used for the election of:

1. A government of national unity (GNU), by a parliament, when cabinet appointments are restricted to members of the parliament;
2. The members of a constitutionally imposed power-sharing executive by elected legislators, as in Northern Ireland or any other post-conflict zone, assuming again that only the legislators may serve in the executive;
3. A majority-coalition cabinet by the parliamentary parties concerned;
4. A shadow cabinet by a party in opposition;
5. The chairs of various committees and sub-committees in parliament or local councils, again by all concerned;
6. A company board and/or a trades union executive by its members;
7. An executive committee by an association at its annual general meeting; or
8. An executive committee by a political party at its annual conference.

Those elected by the matrix vote would have a common rank as member of the cabinet, executive or committee, but each would undertake a different function—the minister of finance or of foreign affairs in government, for example, or the chair-person or treasurer on an executive committee.

If a matrix vote were to be used in the Irish Parliament, *Dáil Éireann*, for the election of a cabinet of 15 ministers (the number in government in Oct. 2009), the ballot paper would be as shown in Table 1. Because the matrix vote is a form of proportional representation, the outcome of such an election would probably if not inevitably be a proportional, all-party, power-sharing coalition cabinet, that is, a government of national unity. The methodology is particularly appropriate for post-conflict societies, not least because it

Table 1. The ballot paper**.** A valid full ballot would contain the names of 15 different TDs (Members of Parliament), one name in each column and one in each row.

	Preferences														
Department of:	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th
Taoiseach, or Prime Minister															
Enterprise, Trade and Employment															
Finance															
Health and Children															
Transport															
Justice, Equality and Law Reform															
Foreign Affairs															
Arts, Sport and Tourism															
Community, Rural & *Gaeltacht* Aff.															
Social and Family Affairs															
Defence															
Environment, Heritage, Local Gov,															
Communications, Energy, Nat. Res.															
Education and Science															
Agriculture, Fisheries and Food															

works without party labels let alone ethno-religious designations.[41]

2 The Matrix Vote—A Short History

The matrix vote was invented by the author. As noted above, it consists of two election counts of one set of ballots. The first election count could be based on any of a number of voting systems for proportional representation, but I consider the most appropriate to be a version of the 'quota Borda system' (QBS) devised by Michael Dummett [3, pp. 283–94; 4, pp. 151–57]. For the second election count, to appoint each of the newly elected to a particular post, I recommend the 'modified Borda count' (MBC—see Section 3.1 below).

The matrix vote was first demonstrated at a cross-community public meeting of over 200 persons, held in Belfast in 1986 under the auspices of the New Ireland Group (NIG).[42] A description of this voting mechanism was published [5, pp. 59–63] to coincide with *The Other Talks*, another NIG cross-party conference on consensus decision-making held in October 1991. The de Borda Institute ran a seminar on electing a power-sharing executive by this methodology in Belfast in 1998, to coincide with the Peace Process. And most recently, an experiment using the matrix vote, a role-playing experiment for electing a GNU, was conducted in Dublin in 2009.

The matrix vote has been adopted by both the NIG and the Northern Ireland Green Party and has often been used for the election of incoming executives at their respective AGMs.

[4141] In Northern Ireland, members of the Assembly must 'designate' themselves as 'unionist', 'nationalist' or 'other', and these designations are used in any consociational votes. In Lebanon, certain governmental appointments are allocated by confessional beliefs, and in Bosnia, some posts are shared according to ethno-religious demarcations.

[42] Despite being some eight years before the ceasefire, this 'experiment in consensus' attracted over 200 persons, including senior figures from both Sinn Féin and the UUP, then known as the Official (now Ulster) Unionist Party. It was successful and a consensus was found. They concluded: 'Northern Ireland to have devolution and power-sharing under a Belfast-Dublin-London tripartite agreement'. It was, as it were, a mini-Belfast Agreement, twelve years ahead of its time.

In addition, it has been used by Mediation Northern Ireland to help solve an industrial dispute, and it has also been demonstrated abroad, for example in seminars in Bulgaria and Germany.

3 The Two Election Counts

The matrix vote is used to elect a fixed number of individuals, n, each of whom is to undertake one of n different functions. In choosing such an executive of n members, each voter in the electorate is permitted to nominate, in his/her order of preference, up to n different individuals, and to propose one of n different posts for each of these nominees. In effect, the voter gives a 1 to his/her 1st preference candidate to be in one particular post, and may give a 2 to his/her 2nd preference candidate to be in another particular post, and so on. As in STV, a vote need contain only a 1st preference in order to be valid.

3.1 The First Election Count

Dummett's QBS (quota Borda system), a variation of which is used for the first election count, is built on two ideas:

1. Representation is given to any sufficiently large set of voters who are 'solidly committed' to a particular set of candidates. The set of voters S is solidly committed to the set of candidates C if every voter in S ranks every candidate in C ahead of every candidate that is not in C [3, p. 282]. The quota, q, that specifies the size that a coalition must be, in order to deserve one representative under QBS is $V/(n + 1)$, rounded up to an integer, where V is the number of voters and n is the number of candidates to be elected [3, p. 284]. The number of representatives that any solid coalition deserves is the smaller of a) the number of voters in the coalition divided by q, rounded down to an integer, and b) the number of candidates in the set to whom the voters are solidly committed.[43]

2. Positions not filled on the basis of solid coalitions are filled by the candidates who have the highest 'modified Borda counts', (MBCs). In a Borda Count (BC), where n is the number of candidates, points are awarded to (first, second … last) preferences according to the rule of either (n, $n - 1$, …, 1) or ($n - 1$, $n - 2$, …, 0). In an MBC with the same number n of candidates, points awarded are (m, $m - 1$, …, 1), where m is the number of candidates that the voter has ranked. In those instances where the voter has cast a full ballot, there is no difference between the two; where the voter has cast a partial ballot, however, the difference can be considerable.[44] The reason I recommend MBC rather than BC is that MBC generates a very strong incentive for voters to rank as many candidates as there are positions to be filled.

In addition to this difference between BC and MBC, there is one other important difference between current rules for the first count of the matrix vote and the QBS rules proposed by Dummett: Instead of providing representation for coalitions that are solidly committed to candidate sets of all sizes, as Dummett proposes, representation based on solid coalitions is provided, in the case of elected bodies of three or four members, only for single candidates and pairs of candidates gaining one or more quotas of 1st and 1st/2nd preferences respectively, while for elected bodies of five or more members, representation based on solid coalitions is provided for single candidates and pairs and triplets of candidates with sufficient top preferences (more details below).

QBS, which is used for the first election count, proceeds by stages, with each stage after the first undertaken only if seats are still unfilled. The limit on consideration of top preference in the Dublin experiment was the simpler one (as if the executive were of only three or four members). Such a count is conducted as follows. In stage i) any candidates receiving a quota of 1st preferences are elected. In stage ii), if a pair of candidates gains 2 quotas of 1st/2nd preferences, then both candidates in that pair are elected.[45] Only

[43] If all voters were in coalitions whose sizes were exact multiples of q, then one too many representatives would be selected, and it would be necessary to choose one at random to be excluded.

[44] In fact, this (m, $m - 1$, …, 1) rule is similar to that which was originally proposed by J-C de Borda [2; 9, p. 197].

[45] A 'pair with 2 quotas' is defined as follows: if x people cast 1st/2nd preferences for Messrs. F and H; if y people cast 1st/2nd preferences for Messrs. H and F; and if $x + y \geq 2$ quotas, then the F/H pair has 2 quotas [6, pp. 41 et seq.].

candidates still unelected are included in any subsequent calculations. In the next stage, iii), seats are awarded to those pairs of candidates gaining 1 quota of 1st/2nd preferences, the actual seat going to the candidate of the pair with the higher MBC score. Finally, in stage iv), any remaining seats are awarded on the basis of MBC scores only. So, while success in stages i) and ii) can be achieved just by achieving the required quantity of top preferences, success in the later stages depends on the candidates' MBC scores, which tend to be highly dependent on cross-party support.

3.2 The Second Election Count

The second election count, conducted by MBC, is concerned with the allocation of successful candidates to positions. For this count, the tellers create a table showing how many MBC points each winning candidate received for each position.

An example is shown in Table 2. The first step in the second count is taken on the basis of the largest cell total. The position represented by the row of this cell is assigned to the person represented by the column of the cell. Next, the second-largest cell total is considered. If this is for the same candidate who received the first position, or if it is for the same position as was assigned to that candidate, then it is skipped, and the third largest total is considered. The count continues, examining the cells in order of decreasing total, and each time a cell is encountered that is for a position that has not been assigned to a candidate and for a candidate who has not been assigned a position, the position is assigned to that candidate. If all the cells with positive points have been considered and not all positions have been filled, the remaining positions are filled by successively awarding the remaining *position* that received the most total points to the remaining *candidate* who received the most total points, until all positions have been allocated.

4 The Dublin Experiment

Because of the parlous state of the Irish economy in 2009, there was much talk about the desirability of a government of national unity (GNU). At the time, however, there was little or no discussion of how such a coalition could or should be chosen. Because negotiations for majority coalition governments, let alone a GNU, tend to be both protracted and problematic, the de Borda Institute decided to conduct a trial to see if, in theory, a parliament could elect a GNU, a proportional, all-party, power-sharing, coalition cabinet, by means of a matrix vote.

If the *Dáil* were to elect such a GNU by this methodology, every TD (*Teachtai Dála*—member of *Dáil Éireann,* the Irish Parliament) would be a candidate for all 15 departments in the cabinet (although, if he/she so wished, any TD could state in advance that he/she did not want to stand for any one, or more, or even all of the ministerial posts). Furthermore, every TD would be able to vote for a cabinet among TDs from all parties in his/her order of preference.

In a QBS election of 15 cabinet members, if all 165 of the TDs (all, that is, except the Speaker) submitted votes, the quota would be 11. A party with more than 7 per cent of the seats in the *Dáil* could expect to win about the same percentage of the executive, so a party with 40 per cent of the seats could realistically hope for 6 of the 15 ministerial positions. Therefore a TD from this party would be well advised, having cast the first 6 or maybe 7 preferences for his/her party colleagues, to cast any lower preferences for those TDs of other parties whom he/she considered suitable likely contenders.

To make the experiment simpler, the Dáil was assumed to contain just 48 TDs, namely, those listed in the appendix, all of whom have achieved a certain degree of prominence in Irish society. The numbers of TDs from the parties were proportional to the strengths of the parties, but the smaller number did mean that independent TDs were excluded. It would have been easier if the experiment had been to elect a government of as few as just 6 ministers, but this would have made it more difficult to demonstrate the proportionality that is so important for a procedure for electing a GNU.

The participants in the experiment were thirty members of the public. They were not asked their party affiliations. In a rotation determined by the sequence in which they

Table 2. The results of the QBS and MBC elections**.**

	Successful TDs															Total points
	MM	*RQ*	*RB*	*JG*	*SC*	*CO*	*BL*	*ND*	*AS*	*DA*	*BC*	*LV*	*EG*	*BS*	*OM*	
Department of:	FF	Lab	FG	GP	FG	SF	FF	FF	FG	FF	FF	FG	Lab	FF	FG	
Taoiseach, or Prime Minister	292		258													550
Enterprise, Trade and Employment		7			181											271
Finance		151					272						16			439
Health and Children									212				4			303
Transport								1				55				266
Justice, Equality and Law Reform			13					236								403
Foreign Affairs			103												176	294
Arts, Sport and Tourism																344
Community, Rural and *Gaeltacht* Aff.				2												209
Social and Family Affairs											5		122			215
Defence										11	197					334
Environment, Heritage, Local Gov.				130						201						375
Communications, Energy, Nat. Res.						89						138				308
Education and Science													36	178		260
Agriculture, Fisheries and Food												7	2			129
QBS success	1st	2nd	3rd	4th	5th	6th	7th	8th	9th tie	9th tie	11th	12th	13th	14th	15th	
Singletons, 1st prefs, totals	17	7	5	5	3	3										
Singletons, quotas of 1st prefs	5+	2+	1+	1+	1	1										
Pairs, double quotas of 1st/2nd prefs	(6)						6									
Pairs, single quotas of 1st/2nd prefs								-	-	-	-	-	-	-	-	
MBC point totals	292	158	374	132	181	89	272	237	212	212	202	200	180	178	176	

The 15 most successful TDs with their party affiliations are shown along the top. Their QBS results and MBC totals are shown at the bottom, in orange, while their MBC cell totals for the various ministerial posts are in the matrix. The column on the right shows the total number of points cast in connection with each portfolio. If the numbers add up horizontally, as they do in the *Taoiseach* row, then no other candidates got any points for this post. If they do not add up, as in the Enterprise, Trade and Employment row, then one or more of the unsuccessful candidates also received some points for this Ministr.

arrived, each of the thirty persons was allocated to a particular party group—*Fianna Fáil* (FF), *Fine Gael* (FG), Labour (Lab), Progressive Democrats (PD), Green Party (GP), or *Sinn Féin* (SF). The first part of the evening was a PowerPoint presentation on the matrix vote and an explanation of the experiment. Each group then split into its own workshop, there to deliberate, with questions on the methodology to the organisers as required, as to how to cast their ballots. The party groups of 3–4 individuals were then given 20, 14, 5, 2, 2 and 1 ballot paper(s) respectively, in direct proportion to current party strengths in the *Dáil*, a total of 44 ballots. (The conduct of the experiment was not affected, therefore, by the actual number of participants.) The second half hour allowed for inter-party talks; this was a fascinating exchange, as groups large and small sought to advance their own interests.

With 44 votes electing a cabinet of 15 ministers, the quota was 3. Thus Labour, with 5 votes, was guaranteed to get 1 person elected. That is, if just 3 of the Labour votes gave a 1st preference for one particular TD, the latter would be successful, albeit in an as-yet-unknown portfolio. FF, meanwhile, with 20 votes, had 6 quotas of 1st preferences, so if the FF group split their 18 votes appropriately, they could get 6 ministers elected; furthermore, if they cooperated with another party, they could use their 2 other votes to get a seventh minister. Alternatively, they could give all 20 of their 1st preferences to one particular TD for the post of *Taoiseach*, (Prime Minister), and thereby all but ensure that this individual would indeed become *Taoiseach*.

There were many possible tactical choices. Each party group could choose whom they wanted to be in the cabinet and who in which department, knowing that if they were the biggest party, they could pretty well guarantee for themselves the most important ministerial post but not necessarily the next most important, but maybe again the third portfolio, and so on. At the same time, they could use any other votes and many lower preferences in negotiations with other party groups.

The intra-group conversations were animated, while the subsequent inter-party negotiations witnessed much hard bargaining. Most groups chose to act in a fairly united way, and many of the FF and FG ballots, for example, followed their own distinct pattern. Because the experiment was conducted in Ireland, where all participants are quite used to the concept of preference voting in elections, the groups were well able to work out how best to use their 1st preferences. How to make the most of their subsequent preferences, however, proved to be more difficult, especially in the limited time available. Furthermore, it was relatively easy for the SF group, which had only one ballot, to decide on its tactics; it was much more difficult for the FF group, with its 20 ballot papers.

FF, the biggest group, decided that they wanted the post of *Taoiseach*, and that Micheál Martin was their candidate. Of the FF votes, 17 had preferences of Martin 1st, Lenihan 2nd, and 1 vote had preferences of Lenihan 1st, Martin 2nd. With their 2 other votes, the FF group came to a deal with SF so that the latter's Caoimhghín O'Caoláin also got a quota of 1st preferences. Most of the FF votes went on to give their 3rd-4th-5th-6th preferences to Noel Dempsey-Dermot Ahern-Brendan Smith-Brian Cowen, so all of these TDs got MBC scores sufficient for ministerial office.

With 14 votes, the FG group had 4 quotas of guaranteed seats and 2 'spare' votes. Five of their 1st preferences were for Richard Bruton; 3 for Simon Coveney; 3 for Ruairí Quinn of Labour; and 3 for John Gormley of the Greens. So Bruton and Coveney were elected in stage i), along with Alan Shatter, Leo Varadkar and Olivia Mitchell in stage iv), on the basis of their MBC scores. Labour's Quinn and the Greens' Gormley got lots of lower-preference support from the other FG votes.

Of their 5 votes, Labour gave 4 of their 1st preferences to Quinn. Quinn thus got 4 Labour plus the above 3 FG 1st preferences and was second in the QBS election. Labour's 2nd, 3rd and 4th preferences went to Bruton, Joan Burton, and Michael D. Higgins.

The PD group used their 2 votes in an attempt to get Mary Harney elected. They tried to do a deal with the FG group, but the latter, it later transpired, reneged. Both of the PD 2nd preferences went to Labour's Quinn and their 3rd preferences to the Greens' Gormley.

The 2 GP votes gave their 1st preferences to Gormley, their 2nd preferences to O'Caoláin (while SF gave Gormley only a 14th preference), their 3rd preferences to the PD's Mary Harney, and most of their lower preferences to FG and Lab.

Finally, the SF group, with only a single ballot, gave its 1st preference to O'Caoláin, most of its other high preferences to FF, and just the odd lower preference to Labour's Eamon Gilmore (12th) and, as already noted, the Greens' Gormley (14th).

5. The Outcome of the Vote

In the QBS election, as shown in Table 2, Martin, Quinn, Bruton, Gormley, Coveney and O'Caoláin all gained a quota of 1st preferences, so they were elected in stage i). In stage ii), the Lenihan/ Martin pair got more than 2 quotas of 1st/2nd preferences, so Lenihan was the seventh person elected. There were no pairs of unelected candidates gaining a single quota of 1st/2nd preferences in stage iii); so all the other elected candidates were chosen in stage iv) on the basis of their MBC scores: Dempsey, Shatter, Ahern, Cowen, Varadkar, Gilmore, Smith and Mitchell.

The second election of the matrix vote—the allocation of the successful TDs to the portfolios shown in Table 3—was determined by portfolio-specific MBC cell totals in the matrix. The highest cell total was 292, for the selection of Martin as *Taoiseach*, and he was appointed to this position. The second highest matrix entry, 272, put Lenihan into Finance. The third gave Justice, Equality and Law Reform to Dempsey. And so on. In this way, 12 TDs were allocated, as shown in grey tint. This left 3 TDs still awaiting appointment and 3 posts unfilled, all shown in pink, but none of these 3 candidates had scored any points for any of these 3 departments. Accordingly, the remaining appointments were made on the basis of the most popular TD (as shown in the orange QBS popularity row at the bottom) gaining that portfolio for which most points had been cast (as shown in the right hand column). The corresponding appointments are indicated

Table 3. The appointments**.**

	Successful TDs															Total points
	MM	*RQ*	*RB*	*JG*	*SC*	*CO*	*BL*	*ND*	*AS*	*DA*	*BC*	*LV*	*EG*	*BS*	*OM*	
Department of:	FF	Lab	FG	GP	FG	SF	FF	FF	FG	FF	FF	FG	Lab	FF	FG	
Taoiseach, or Prime Minister	292		258													550
Enterprise, Trade and Employment		7			181											271
Finance		151					272						16			439
Health and Children									212				4			303
Transport								1				55				266
Justice, Equality and Law Reform			13					236								403
Foreign Affairs			103												176	294
Arts, Sport and Tourism																344
Community, Rural and *Gaeltacht* Aff.				2												209
Social and Family Affairs											5		122			215
Defence										11	197					334
Environment, Heritage, Local Gov.				130						201						375
Communications, Energy, Nat. Res.						89						138				308
Education and Science													36	178		260
Agriculture, Fisheries and Food												7	2			129
MBC point totals	292	158	374	132	181	89	272	237	212	212	202	200	180	178	176	

The MBC scores in the matrix are taken in descending order: 292 is the highest; 272 is 2nd; 236 is 3rd, and each of the top cell totals are ranked in this way, as described in the text, and as shown in tints of grey. A high cell total is not ranked if it has been superseded by another higher cell total. Thus while *RB* gets 258 points for the post of *Taoiseach*, that post is no longer vacant; such superseded cell totals are shown in yellow. The grey squares thus indicate which TDs have been allocated to which posts. The pink indicates those TDs, and those posts, which cannot be allocated on the basis of cell entries. And turquoise portrays those appointments for which these (3 pink) TDs received scores of 0.

in Table 3 in turquoise, while Table 4 shows the outcome.

6 Analysis

The overall outcome was as one might have expected from a reliable PR electoral system: FF, 6 seats; FG, 5; Lab, 2; PD, 0; GP, 1; and SF, 1.

There were some tactical disappointments. For example, FG tried to get Richard Bruton appointed as *Taoiseach* but his 258 points were trumped by the 292 points of Micheál Martin from FF. As a second option, the FG group hoped that Bruton would become Minister of Foreign Affairs, for which he got 103 points, but here too he lost, this time to his own party colleague, Olivia Mitchell, with 176 points. In like manner, the GP group lost the Environment, Heritage and Local Government Department, for while John Gormley got 130 points for this portfolio, Dermot Ahern of FF received 201 points. As it was, Gormley was appointed to Community Rural and *Gaeltacht* Affairs with only 2 points, hardly a ringing endorsement.

Perhaps the biggest weakness of the matrix vote relates to those ministers who were appointed with scores of 0: Ruairí Quinn, Richard Bruton and Caoimhghín O'Caoláin all became ministers in departments for which they had received no points at all. O'Caoláin, with only 89 points in total, could hardly object; but supporters of Quinn and Bruton, 2nd and 3rd in the QBS election, with total MBC scores of 158 and 374 respectively, had cause to be critical.

Table 4. The Outcome.

	Successful TDs															Total points
	MM	*RQ*	*RB*	*JG*	*SC*	*CO*	*BL*	*ND*	*AS*	*DA*	*BC*	*LV*	*EG*	*BS*	*OM*	
Department of:	FF	Lab	FG	GP	FG	SF	FF	FF	FG	FF	FF	FG	Lab	FF	FG	
Taoiseach, or Prime Minister	292 1st															550
Enterprise, Trade and Employment					181 7th											271
Finance							272 2nd									439
Health and Children									212 4th							303
Transport			0 14th													266
Justice, Equality and Law Reform								236 3rd								403
Foreign Affairs															176 9th	294
Arts, Sport and Tourism		0 13th														344
Community, Rural and *Gaeltacht* Aff.				2 12th												209
Social and Family Affairs													122 11th			215
Defence											197 6th					334
Environment, Heritage, Local Gov.										201 5th						375
Communications, Energy, Nat. Res.												138 10th				308
Education and Science														178 8th		260
Agriculture, Fisheries and Food						0 15th										129
MBC point totals	292	158	374	132	181	89	272	237	212	212	202	200	180	178	176	

This table shows the final cabinet, with each appointment shown in grey, with both the candidate's MBC cell total and his/her ranking in these appointments. Only information pertaining to the final cabinet is shown.

One of the unfilled appointments was the Department of Arts, Sport and Tourism, for which 344 points had been cast. Of these, the highest individual cell total of 165 points was for Pat Carey to take on this portfolio. But in the QBS election, Carey lost, albeit by a narrow margin: he was sixteenth. But why appoint someone with a score of 0, when the consensus of those voting appeared to support another?

Meanwhile, in the Department of Transport, a total of 266 points had been cast. Of these, Phil Hogan got a cell total of 146 and was eighteenth in the QBS election; the other candidate with a reasonable score for this Department was Leo Varadkar with 55 points, but his total MBC was 200 and he was already in the cabinet in the post of Communications, Energy and Natural Resources. So should Hogan have got the Transport job?

In a nutshell, was it right for Quinn and Bruton to get these two departments, with 0 points, when, in the consensus of those voting, others were more suitable? Should the rules be changed to allow for the appointment of ministers without portfolio, so that these two departments would be given to Carey and Hogan and the cabinet would be expended to 17 members? If this same logic were to be applied to the post of Agriculture and Rural Development, then Eamon Ryan would have been similarly rewarded, but he had a mere 69 points for that Department, and in the QBS election he was twenty-third in order of popularity. So would this mean a cabinet of 23 members, with a total of 8 without portfolio?

As explained below, if there were a real *Dáil* election with 165 voters, an outcome with such zero-point appointments would be unlikely. Furthermore, in any electoral system, there will always be winners and losers, and some of the latter might feel they have been 'pipped at the post'. Nevertheless, any feelings of disappointment with the outcome will usually apply not to the most popular figures, but to the less popular TDs, those who came 16th and lower in the QBS election and to those ministerial posts receiving smaller totals of points per portfolio.

7 The Potential Role of the Matrix Vote

The chances of the matrix vote being adopted by society at large, in business, trade unions and community associations, is probably fairly small, at least until such time as programs for electronic voting are more readily available. In political circles, however, prospects are better because the matrix vote allows all participants (e.g., every member of parliament) to seek selection (e.g., for the cabinet) by appealing to their fellow participants, and it allows all to have equal influence on the outcome, without resort to party labels, let alone sectarian or other designations. One disadvantage, in the view of some politicians, might be that it is quite difficult to predict the outcome, but such a property should really be regarded in a positive light. The more unpredictable an electoral system, the more difficult it is to dominate and control.

Another disadvantage, many will argue, is that it will allow extremists to exercise power: the likes of the Freedom Parties in Austria and the Netherlands. This criticism is somewhat off-target, however, for both of these parties have already exercised more than their fair share of power; the former joined the People's Party in a majority coalition in 2000, and the latter is currently supporting the Dutch administration [8]. With a matrix vote, any small party (and any big party, for that matter) would exercise influence and power only according to its proportional due.

In a majoritarian system, a small party—or even a single 'king-maker' independent—can occasionally wield excessive power. With all-party power-sharing, however, a small party should exercise only its fair share of power. It is interesting to note in this regard that some people oppose the introduction of PR electoral systems because, they say, it might allow extremists into parliament. The danger, however, lies more in the particular form of PR that is chosen. In Austria and the Netherlands, where extremists have indeed managed to achieve exaggerated prominence, party-list forms of PR are used. A preferential form of PR, such as STV or QBS, would provide a more accurate reflection of their support. Elections in Northern Ireland show that persons who vote for extreme parties often fail to cast any lower preferences for other parties, unlike those who support one or other of the more moderate parties, who often give lower preferences to candidates of 'neighbouring' parties, [7, p. 207]. This would tend to reduce the number of extremists elected. In the 2011

Assembly and local elections in Northern Ireland, for example, the Alliance Party, which is arguably the opposite of extremist, has done rather well.

Despite its benefits, the chances of the matrix vote being introduced in any democracy are probably minimal, not least because reform of the present structures depends, in large measure, on the cooperation of those who benefit from the current rules. The chances of persuading any government in general, or the *Dáil* in particular, to adopt the matrix vote are therefore slim. Before the February 2011 general election, FG was unlikely to agree to such a procedure for they knew FF was unpopular and over-represented. And now that FG has had such a successful election, it is even less likely to favour the idea of a GNU. Admittedly, it failed to gain an overall majority, so despite having a number of differences, not least on economic policies, it has formed a majority coalition with the Labour Party. At some future date, therefore, it could be open to using the matrix vote as a means by which the two parties might reshuffle a coalition cabinet.

Many Members of the Legislative Assembly of Northern Ireland are committed to power-sharing but opposed to sectarian or other designations. Since the matrix vote procedure is proportional and works without any labels, it might be favoured if those concerned were more aware of its existence and/or if the matrix vote were already in widespread use in society at large, for such situations as associations' AGMs.

Among the advantages of the matrix vote are: it allows a relatively large number of individuals to be eligible for election while allowing those who wish to opt out to do so; it provides a strong incentive for voters to cast full ballots of their preferences; it encourages cooperation rather than division; it is transparently inclusive; and it ensures a proportional result.

8 Possible Alternatives

Since the matrix vote could lead to the appointment of persons who, though popular overall, have no particular talents for the departments to which they have been appointed, there is at least one possible variation that might be attractive: parliament could elect the members of its all-party cabinet by PR (and the method I would recommend would indeed be QBS or at least STV). Then parliament could conduct a second vote to appoint each of these elected candidates to a department. In the Irish case, this would mean a QBS election with up to 165 candidates—all the TDs other than the Speaker—for the 15-member cabinet; and then a 'second count' MBC matrix vote with just these 15 to see who would be appointed to each ministry. Such a procedure would have the additional advantage that all votes in the second count would be for candidates who would actually be assigned to a particular portfolio.

The disadvantage of such a two-round procedure is that a lot of information would thereby be lost. When the matrix vote is conducted as it was in the above experiment, the levels of support received by various candidates, even by those not elected to the cabinet, were nevertheless apparent.

It is always possible of course, that those concerned will not use the matrix vote to its full potential, that certain persons will cast preferences only for colleagues from their own party, that in post-conflict scenarios, some persons may not vote for an individual because of the latter's ethno-religious identity, or simply because of their gender. That said, it is nevertheless true that most would probably be tempted to make full use of the power that a matrix vote would give them. Just as any member of a football club might rejoice if given the opportunity to help select a full team, and doubtless he/she would choose a full eleven players in all, each most suited (in that fan's opinion) to the position allocated, so too most members of parliament would probably be more than keen to vote for a full cabinet, if allowed to do so.

9 Conclusion

There are, indeed, possible weaknesses to the matrix vote. Given i) the task for which it is designed; ii) the fact that it is based on two electoral processings of the preferences cast; and iii) that Arrow's 'impossibility theorem' applies to every voting system [1]; some weaknesses are only to be expected. The main one encountered in the trial—the appointment of ministers to departments for which they had received no support—is less likely if the number of those voting is larger. Thus, in real life, when all parties in the *Dáil* would have a fair understanding of the workings of the matrix vote, and if (nearly) all 165 non-Speaker members cast full ballots of 15 preferences, the chances of any TD being appointed to a department for which he/she had no support would be minimal. This is all the more true since, under such a form of governance, the bigger parties would be highly likely to engage in talks, just as they did in Germany in 2005, prior to forming a grand coalition. Even in the divided society of Northern Ireland with its d'Hondt system, 'departmental allocations were agreed in advance' [10, p. 186]. With a matrix vote, not least because, as explained above, the voting system itself encourages full ballots and cross-party voting, the prospects of such inter-party cooperation would be even greater. So the chances of a popular TD or MP finding him/herself appointed to a department with a score of 0 would be tiny.

In a majoritarian milieu, parties might not talk to each other. If the rules provided for cooperation, however, then the atmosphere might change. Ideally, a power-sharing executive would commit to taking its decisions by consensus. Politicians are always quick to understand the characteristics of any voting procedure. In STV, for example, because of its quota element, parties rarely nominate more candidates than they think will get elected. QBS shares this characteristic. Similarly, if the matrix vote were to be adopted, the nature of its procedures would almost certainly mean that politicians and parties would work in a more inclusive way

10 Acknowledgements

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About the Author

As director of the de Borda Institute, Peter Emerson has worked in many conflict zones—in Northern Ireland, the Balkans, the Caucasus, and East Africa—promoting consensus voting, including the matrix vote, in power-sharing structures of governance. His latest works include *Designing an All-Inclusive Democracy*, Springer-Verlag, 2007; *Party Politics in the Western Balkans* (co-ed.), Routledge, 2010; Consensus Voting and Party Funding, *European Political Science*, Vol. 9, № 1, March

2010, pp. 83–101; Proportionality without Transference, *Representation,* Vol. 46, № 2, July 2010, pp. 197–209; The Next Scottish Referendum, *Scottish Affairs*, № 73, Autumn 2010, pp. 36–54; and *Defining Democracy*, Springer-Verlag, 2011 (forthcoming).

Appendix: The 48 TDs, listed alphabetically by surname, with identifying initials for the winners.

Fianna Fáil (FF)	*Fine Gael* (FG)	Labour
Dermot Ahern (*DA*)	Richard Bruton (*RB*)	Joan Burton
Barry Andrews	Simon Coveney (*SC*)	Eamon Gilmore (*EG*)
Áine Brady	Jimmy Deenihan	Michael D. Higgins
Dara Calleary	Olywn Enright	Liz McManus
Pat Carey	Charlie Flanagan	Ruairí Quinn (*RQ*)
Mary Coughlan	Brian Hayes	Pat Rabbitte
Brian Cowen (*BC*)	Phil Hogan	Róisín Shortall
Noel Dempsey (*ND*)	Enda Kenny	7
Sean Haughey	Olivia Mitchell (*OM*)	
Tony Killeen	Denis Naughten	'Progressive Democrats' (PD)
Brian Lenihan (*BL*)	Fergus O'Dowd	Mary Harney
Conor Lenihan	James Reilly	Finian McGrath*
John Moloney	Michael Ring	2
Micheál Martin (*MM*)	Alan Shatter (*AS*)	
Éamon Ó Cuív	William Timmins	Green Party (GP)
Willie O'Dea	Leo Varadkar (*LV*)	John Gormley (*JG*)
Batt O'Keefe	16	Eamon Ryan
Peter Power		2
Dick Roche	*Sinn Féin* (SF)	
Brendan Smith (*BS*)	Caoimhghín O'Caoláin (*CO*)	
20	1	

*Finian McGrath is actually an independent TD.

Another Note on the Droop Quota and Rounding

Svante Janson
svante.janson@math.uu.se

Abstract

The Droop quota is traditionally rounded up to the next integer. It has been pointed out by Lundell and Hill that this can have negative consequences. We give two further examples of such consequences, showing that rounding the Droop quota can cause violations of monotonicity and proportionality. This supports the conclusion by Lundell and Hill that it is better to use the exact Droop quota without rounding.

Keywords: Droop quota, rounding

1 Introduction

Consider an election where v votes are cast and there are s seats to be filled. The standard Droop quota, as defined by Henry Droop [1], is $v/(s+1)$ rounded up to the nearest strictly larger integer, i.e., $\lfloor v/(s+1)\rfloor+1$. This is the quota used in many STV elections, but it has later been realized that there is no compelling reason to use an integer quota except in versions of STV that only transfer whole votes (perhaps in the traditional way by physically moving ballots by hand between different stacks). Thus the exact (unrounded) Droop quota $v/(s+1)$ is also used in some modern versions of STV, for example the ERS rules [3].

The two versions of the Droop quota were compared by Lundell and Hill [2], who concluded that the exact version generally is better. (To avoid some problems, they then also recommended that candidates are elected only when they *exceed* the quota, a rule suggested by Mann [4].) The purpose of the present note is to add two further reasons for using the exact Droop quota whenever possible.

Remark 1. As discussed by Lundell and Hill [2], the terminology is varying, with other names sometimes used for the exact version. (For example "NB quota" in [4].) I agree with them on using "Droop quota" for both versions; I will distinguish the two versions by calling them "exact Droop quota" and "rounded Droop quota". (Strictly speaking, the "rounded" version is not rounded, since it is increased even when the exact quota happens to be an integer.)

Remark 2. In practice, the calculations are usually done to a fixed number of decimal places, such as 2 for the ERS rules [3]. (Similarly, the Irish senate rules, where each vote is counted as 1000, may be regarded as doing calculations to 3 decimal places.) In such cases, even the exact Droop quota is rounded (usually upwards) to the used precision. I will disregard this and assume that calculations are done exactly, for example by rational arithmetic or by using a sufficiently large precision, preferably with guard digits. (This is discussed further by Lundell and Hill [2].)

In my opinion, if the result of an election depends on the chosen accuracy of the calculations, only the result that agrees with exact (rational) calculations can be defended.

2 Monotonicity

Monotonicity means that increased support cannot harm a candidate. It is well-known that monotonicity can fail with STV: in some situations, a candidate may lose a seat by getting additional support (either by getting additional votes or by moving up on some ballots, perhaps to become the first preference). Woodall [5] discusses several different cases and examples; see also Woodall [6] for a detailed discussion of one example. To use the rounded Droop quota adds further possibilities of violations of monotonicity (of Woodall's type *mono-add-plump* and *mono-add-top* [5]): a candidate or party can lose a seat by attracting a new voter. Example:

(5 seats)

500 ABCDE

99 F

The rounded Droop quota is $\lfloor 599/6 \rfloor + 1 = 100$, and A, B, C, D, E are elected. Suppose now that the ABCDE party gets a new voter, raising their vote to 501. Then the quota becomes $\lfloor 600/6 \rfloor + 1 = 101$; A, B, C and D are elected but the surplus transferred to E is only 97, so E is eliminated and F is elected to the final seat. (With the exact Droop quota, A, B, C, D and E are elected in both cases, with votes strictly exceeding the quota.)

For simplicity, this example uses a party (coalition) with all its voters voting in the same way (as is approximately the case in Australian Senate elections), but note that the result remains the same if the ABCDE voters vote for these candidates in different orders of preference (except that someone else than E may be the one losing a seat). In particular, adding a single ballot E to the example above would be just as bad for E.

The reason for this counterintuitive behaviour is that the quota increases in steps of 1 (or not at all); in this example, the exact Droop quota increases (from 599/6 to 600/6) by a factor 1/599, but the rounded Droop quota increases by a factor 1/100. The new vote increases the votes for party ABCDE by a factor 1/500. Thus, using the exact Droop quota, the ratio *votes/quota* goes up, as it always does, but with the rounded Droop quota, the ratio goes down in this example because the quota increases by a larger factor than the number of votes for ABCDE. In this example, the effect is that one seat is lost.

Note that although rounding looks innocuous when the number of votes is large, the problem is not limited to small elections. For example, we can modify the example above to 5,000,000 and 999,999 votes with the same result.

3 Homogeneity

Homogeneity means that the result only depends on the proportion of ballots of each possible type [5]. STV with the exact Droop quota is obviously homogeneous. Woodall [5] regards STV as homogeneous; he notes that finite precision calculation might give violations but sees this as a minor practical problem. I agree as long as the exact Droop quota is used, but when the rounded Droop quota is used as a matter of principle (perhaps out of tradition), I think that it is justified to regard STV as non-homogeneous. Example:

(9 seats)

71 ABCDEFG
30 XYZ

The rounded Droop quota is $\lfloor 101/10 \rfloor + 1 = 11$, so first A, B, C, D, E, F, X, and Y are elected, leaving G with 5 transferred votes and Z with 8; thus the final seat goes to Z and the ABCDEFG party gets 6 seats. If all votes are multiplied by 10, then the quota becomes $\lfloor 1010/10 \rfloor + 1 = 102$, and at the end G has 98 votes against Z with 96; thus G takes the final seat, giving the ABCDEFG party 7 seats. (This is also the outcome with the exact Droop quota, in both cases. It is further the only outcome consistent with the *Droop proportionality criterion (DPC)* in [5]; for another example of DPC violation with the rounded Droop quota see [2].)

Again, the effect exists also for large elections; we can modify the example to 7,000,001 and 3,000,000 votes.

4 Conclusion

Failure of monotonicity is unfortunately an unavoidable problem for STV. To use the rounded Droop quota adds to this problem. While the added cases might be of minor practical importance, it seems better to reduce the problem as much as possible by using the exact Droop quota.

The property of homogeneity is perhaps less important, but it is certainly desirable and again this is an argument for using the exact quota.

Note that in practice, the exact and rounded Droop quotas usually give the same result, especially in large elections. But in the cases where the difference matters, the exact quota seems to be the better choice; see Lundell and Hill [2] for further examples and discussions. I thus agree with the conclusion of Lundell and Hill [2] that the exact quota should be used when possible.

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About the Author

Svante Janson is professor of mathematics at Uppsala University, Sweden and a member of the Swedish Royal Academy of Sciences. His research is mainly in probability theory and combinatorics. He also has an interest in election methods, in particular their mathematical properties, and has been consulted by the Swedish Election Review Board concerning some appeals to the general election of 2010.

Review – Voting Theory for Democracy

Markus Schulze
markus.schulze@alumni.tu-berlin.de

Abstract

Mathematica is the most important information visualization software. It is proprietary software developed by Wolfram Research. In the book *Voting Theory for Democracy* (323 pages) [1], Thomas Colignatus describes and motivates his voting software add-on (*Economics Pack*) for *Mathematica*. This book is also intended to be a primer in voting theory.

1 Introduction

Section 1 (19 pages) explains Condorcet's paradox and Arrow's theorem. Unfortunately, there are no formal definitions for *preferences*, *orderings*, *single-seat elections*, etc.

I disagree with the author's interpretation of Arrow's theorem. For example, the author writes (page 31): "It is sometimes thought that all problems in voting are caused by Arrow's theorem. This however is a misunderstanding. The problems in voting are not caused by Arrow's theorem but by the possibility of cheating." However, it has been shown by Gibbard [2] and Satterthwaite [3] that it is a direct consequence of Arrow's theorem that all preferential single-seat election methods, that are Paretian and non-dictatorial, are vulnerable to "cheating" (strategic voting).

Furthermore, the author argues that Arrow's theorem is unreasonable because candidates always have to cast an eye not only on all the other candidates, but also on all those politicians who might declare their candidacy at a later time. Therefore, for the concrete campaign strategies of candidate *a*, it doesn't matter whether politician *b* has already announced his candidacy or might do this later. In the words of the author (page 30): "In voting, the relative positions of two candidates might depend upon the budget [that is, the set] of available candidates." However, in my opinion, the author only moves the problem of *irrelevant alternatives* from asking, whether *b* is a *declared* candidate, to asking, whether *b* is an *available* candidate. When *b*, who didn't announce his candidacy, dies and when (in reaction to this change of the pool of available candidates) the other candidates change their positions and when this leads to a change of the final winner, this is still an undesirable event.

Section 2 (2 pages) explains the installation process for the *Economics Pack*. **Section 3** (26 pages) explains the possible formats for the input for the programs to calculate the winners of the different single-seat election methods.

Sections 4 and 5 (72 pages in total) explain the possible visualizations of the input (e.g. pairwise digraphs, Black diagrams, Saari diagrams). Furthermore, they explain all the single-seat election methods whose winners can be calculated with the *Economics Pack*: plurality voting, top-two runoff, Borda, Nanson, and the "Borda Fixed Point" method.

The author's use of some terms differs significantly from their use in the scientific literature. This leads to misunderstandings when, for example, the author concludes that "plurality voting can violate Pareto optimality" (page 70). The author also criticizes the Borda count for violating "Pareto optimality".

Section 6 (24 pages) discusses strategic voting and the no-show paradox. The author presents some examples. But unfortunately, he doesn't present general results.

Section 7 (42 pages) describes the Elo rating system (a method for ranking the relative skill levels of players in head-to-head games; e.g. chess) and the Rasch model (a method for ranking students according to their performance in psychological tests). There is no analysis of these schemes. Without any analysis, the author reaches the conclusion that these schemes are also suitable for public elections.

Section 8 (31 pages) tries to estimate the cardinal utilities of the voters. **Sections 9 and 10** (71 pages in total) discuss Arrow's theorem. At one point, the author "solves" Arrow's theorem by rejecting *independence of irrelevant alternatives* as unreasonable. At another point, the author "solves" Arrow's theorem by keeping the election method undefined in the case of circular ties.

2 The "Borda Fixed Point" Method:

A serious problem of this book is that the author spends too much time introducing his own pet method: the "Borda Fixed Point" (BFP) method. This method has neither been published nor adopted somewhere. Even this book doesn't contain a proper analysis of this method. So why should we be interested in software to calculate the winner of the BFP method?

The *Borda complement* BC[*x*] of candidate *x* is that candidate who would be the Borda winner if candidate *x* didn't run. Candidate *x* is a *Borda Fixed Point candidate* if he pairwise beats BC[*x*]. The *Borda Fixed Point winner* is the winner of a Borda count among all Borda Fixed Point candidates.

The basic idea of the BFP method is that, when candidate *x* is added to the pool of candidates, then candidate *x* should be able to win only by being a better candidate and not simply by the fact that, by his addition to the pool of candidates, this pool is perturbed in such a manner that candidate *x* happens to be chosen by the used election method. The author calls this the "proposal-versus-alternative approach". A new candidate should be able to win only if he is an "improvement" from the original winner (i.e. only if he pairwise beats the original winner).

The author claims that the BFP method satisfies the proposal-versus-alternative condition. But the following examples show that it doesn't.

Example 1

51 *abcde*
49 *cdeba*

We get BC[*a*] = *c*. *a* pairwise beats *c*. Therefore, *a* is a BFP candidate.
We get BC[*b*] = *c*. *b* pairwise beats *c*. Therefore, *b* is a BFP candidate.
We get BC[*c*] = *d*. *c* pairwise beats *d*. Therefore, *c* is a BFP candidate.
We get BC[d] = *c*. *d* doesn't pairwise beat *c*. Therefore, *d* is not a BFP candidate.
We get BC[*e*] = *c*. *e* doesn't pairwise beat *c*. Therefore, *e* is not a BFP candidate.
Now, the Borda count is applied to the BFP candidates:

51 *abc*
49 *cba*

The winner of this Borda count is *a*. Therefore, the BFP winner is *a*.

Example 2

51 *afbcde*
49 *cdefba*

We get *BC*[*a*] = *c*. *a* pairwise beats *c*. Therefore, *a* is a BFP candidate.
We get BC[*b*] = *c*. *b* pairwise beats *c*. Therefore, *b* is a BFP candidate.
We get BC[*c*] = *f*. *c* doesn't pairwise beat *f*. Therefore, *c* is not a BFP candidate.
We get BC[*d*] = *f*. *d* doesn't pairwise beat *f*. Therefore, *d* is not a BFP candidate.
We get BC[*e*] = *f*. *e* doesn't pairwise beat *f*. Therefore, *e* is not a BFP candidate.
We get BC[*f*] = *c*. *f* pairwise beats *c*. Therefore, *f* is a BFP candidate.
Now, the Borda count is applied to the BFP candidates:

51 *afb*
49 *fba*

The winner of this Borda count is *f*. Therefore, the BFP winner is *f*.

Thus the newly added candidate *f* changes the BFP winner from candidate *a* to candidate *f* without pairwise beating candidate *a*.

The author claims that the BFP method satisfies the majority criterion. But example #2 shows that it doesn't.

Furthermore, there are other single-seat election methods (e.g. the Kemeny-Young method [4] and Tideman's ranked pairs method [5]) where a newly added candidate x can win only if he pairwise beats that candidate who would be elected if candidate x didn't run. So the problem that Colignatus addresses has already been solved in the scientific literature.

Furthermore, I don't consider the proposal-versus-alternative condition important because, when the number of candidates is increased from N to $N+1$, then the newly added candidate is always chosen in $1/(N+1)$ of all profiles. Therefore, instead of trying to minimize the number of profiles where the newly added candidate wins, one should rather try to minimize the number of profiles where the newly added candidate x changes the winner from candidate y to some other candidate $z \notin \{x, y\}$.

3 Summary:

This book is not suitable as a primer in voting theory because (1) this book contains too many errors, (2) the author spends too much time on his own pet method, and (3) in too many cases the author's use of terms differs too much from their use in the scientific literature. The author addresses too many topics; but he doesn't address them properly and thoroughly. The main problem of this book is the lack of formal definitions. (For example: The fact, that the author can "solve" Arrow's theorem by keeping the election method undefined in some cases, is only possible because he didn't give a formal definition for election methods.) The author's criticism of Arrow's theorem (which covers about one fourth of this book) is just mumbo-jumbo.

4 References

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[3] Mark A. Satterthwaite, "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions", *Journal of Economic Theory*, volume 10, issue 2, pages 187–217, 1975.

[4] H. Peyton Young, "Condorcet's Theory of Voting", *American Political Science Review*, volume 82, number 4, pages 1231–1244, 1988.

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